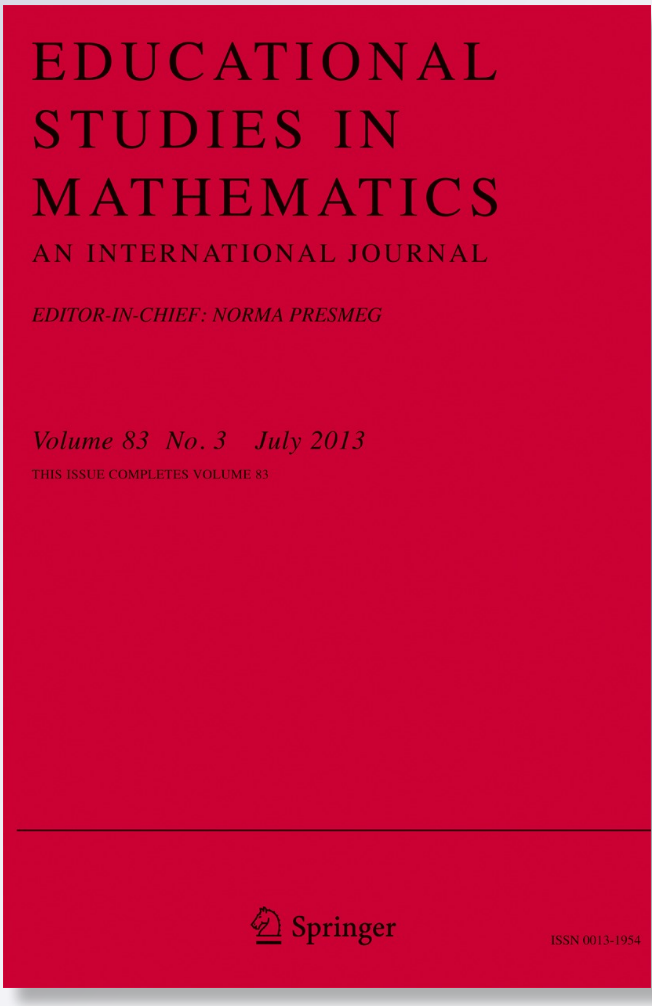
The co-construction of learning difficulties in mathematics—teacher–student interactions and their role in the development of a disabled mathematical identity

Einat Heyd-Metzuyanim

Educational Studies in Mathematics
An International Journal

ISSN 0013-1954
Volume 83
Number 3

Educ Stud Math (2013) 83:341-368
DOI 10.1007/s10649-012-9457-z



Springer

Educ Stud Math (2013) 83:341–368
DOI 10.1007/s10649-012-9457-z

The co-construction of learning difficulties in mathematics—teacher–student interactions and their role in the development of a disabled mathematical identity

Einat Heyd-Metzuyanim

Published online: 29 December 2012

Abstract Leaning on a communicational framework for studying social, affective, and cognitive aspects of learning, the present study offers a new look at the construction of an identity of failure in mathematics as it occurs through teaching–learning interactions. Using the case of Dana, an extremely low-achieving student in 7th grade mathematics, I attempt to unearth the mechanisms of interaction between Dana and myself, her teacher, that instead of advancing Dana, perpetuated her failure. Through examining the interactional routines followed by Dana and me, I show how Dana's deviations from normative routines resulted in my identification of Dana as "clueless" in mathematics. This identification, shared both by Dana and by me, was accompanied by adherence to ritual rule following that did not enable Dana's advancement in mathematical discourse. This case points to the need to re-examine permanent difficulties in mathematics in light of the reciprocal nature of such difficulties' (re)construction in teaching-and-learning interactions.

Keywords Learning difficulties · Social interaction · Identity · Communicational framework

1 Introduction

Why do certain students seem to be unable to learn mathematics? The most prevalent views these days talk about *mathematical disabilities* or *dyscalculia* (Butterworth, 2005; Geary & Hoard, 2005; Ginsburg, 1997), a basically individual, most probably inherent defect in certain students' intellectual abilities that prevents them from learning mathematics or from advancing their mathematical thinking in a normal manner.

In the present study, I shall argue that social and affective processes may be responsible for a *collective* formation of failure to learn mathematics. I shall do this by focusing on the minute-to-minute interactions between a student and a teacher that form part of a constrained path of learning, leading the student to repeated failure and ritual rule following without much chance of advancement to a more agentive, explorative, and skillful performance.

E. Heyd-Metzuyanim (✉)
The Technion - Israel Institute of Technology, Hagalil St. 14, Lapid, Israel
e-mail: einat.metz@gmail.com

The co-construction of failure, which is manifested in an ever-growing disparity between the students' current mathematical skills and those that are expected of her, will be shown through the analysis of Dana's case. Dana was a 7th-grade student with extremely low achievements in mathematics, who despite our mutual effort (hers and mine, her teacher), did not advance in her mathematical skills after 5 months of instruction. In what follows, I shall perform a micro-analysis of the teaching–learning interactions between Dana and me, to show how they were influenced by the identity of failure that was co-constructed by Dana and by me, and how these interactions constrained Dana from advancing her mathematical skills.

2 Theoretical background

2.1 Learning difficulties in mathematics

Most theories that attempt to explain mathematical difficulties view mathematics as an innate or stable *ability* and therefore attribute mathematical difficulties to *dis*ability, most probably caused by early developmental or neurological deficits (Butterworth, 2005; Shalev et al., 2001). This view has gained considerable popularity not only in the research community, but also in educational settings and in the public discourse. And yet, there is no comprehensive theory, to this date, of mathematical learning disabilities that is generally endorsed by the research community. This may be in part because of disagreement regarding the basic phenomena under investigation, expressed in the fact that the definitions of mathematical learning disabilities and dyscalculia have been a constant topic of debate (Kosc, 1974; Mazzocco, 2005). Above all, the problem with the learning disability construct is the inability to differentiate between difficulties that signal *a stable disability* in mathematics and those that are a result of deficient teaching experiences or lack of sufficient exposure (González & Espínel, 1999; Mazzocco & Myers, 2003).

In reaction to this debate, researchers have developed the idea of *response to intervention* (Fletcher & Vaughn, 2009; Fuchs et al., 2005) as a criterion for determining disability from difficulties. However, these studies rarely refer to *what* kind of intervention, or instruction, is necessary to prove that the student is unable to learn mathematics. In practice, students who are placed in low level sets (including those diagnosed as mathematically disabled) often receive different instruction from high-set students (Eisenmann & Even, 2011; Linchevski & Kutscher, 1998; Solomon, 2007). In fact, several researchers explicitly recommend different methods of instruction to mathematically learning disabled students (Jones, Wilson, & Bhojwani, 1997; Montague, 2007).

Another more sociological or anthropological view suggests that mathematical failure, and learning disabilities in general, may not be only an individual affair, but a collective accomplishment of the student, the teacher, and the students' social milieu (Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005; Black, 2004a; Lange, 2009; McDermott, 1993; Solomon, 2007). For instance, Solomon has claimed that top set and low set identities differ significantly because of the different classroom practices that are customary in them. Black has shown that this inequality is not only present in different ability groups, but also in the same class, where certain students are privileged according to teacher expectations of ability and cultural capital. And yet, studies concerning the differential construction of mathematical identities have not inquired into the actual learning–teaching interactions in which low-achieving students participate, and into how these interactions may be responsible for these students' under-developed mathematical skill.

Students' difficulties with mathematics have been studied by another stream of research that may be broadly termed "affect in mathematical learning" (Leder & Forgasz, 2006; McLeod, 1992). This research has concentrated on how certain beliefs (such as self-efficacy beliefs, or beliefs about what mathematical problems solving entails) might hinder effective mathematical learning (Pajares & Graham, 1999; Schoenfeld, 1989). Though such beliefs and emotions (especially negative ones such as math anxiety) have been long acknowledged as significant in mathematical learning (DeBellis & Goldin, 2006; Evans, Morgan, & Tsatsaroni, 2006; Hembree, 1990; Ma, 1999; Pajares & Graham, 1999) little is known about *how* these affective phenomena actually interact with the development of learning difficulties. One reason for this gap in our knowledge may be that research on affect in mathematics has been done mainly with questionnaires and interviews, which are not suitable for studying the minute-to-minute process of learning. A deeper reason may be that the underlying theoretical assumptions about the dual nature of affect and cognition have dictated different methodological tools for studying these phenomena.

It seems, therefore, that in order to study affect, cognition and social interactions in mathematical learning, the first step would be to create a unifying conceptual framework for studying these three aspects of learning. The sociocultural view, which often looks at cognition as it is socially constructed, forms a promising starting point for such a unified conceptual framework. In fact, several researchers have already started to use sociocultural (or socio-historical) frameworks to study emotional aspects of mathematical learning (Evans et al., 2006; Roth & Radford, 2011). Of all the sociocultural theories, the commognitive framework proposed by Sfard (2008) seems particularly effective for the unification goal as it has already gone a long way in providing operational tools both for studying the development of mathematical discourse (Sfard & Lavie, 2005; Sfard, 2007) and for studying identity in mathematical learning (Sfard & Prusak, 2005). In a recent paper, Heyd-Metzuyanim and Sfard (2012) have elaborated on Sfard and Prusak's work by presenting conceptual tools for studying the process of identity construction, that is—*identifying*. In the present work, I shall shortly introduce this framework and elaborate it to include tools for analyzing teacher–student interactions. Those will be used to explain how disabled mathematical identities are perpetuated through such routine interactions.

2.2 The commognitive theory of mathematical learning

The communicational (commognitive) framework belongs to participationist theories that view learning as becoming a participant in a community (Lave & Wenger, 1991) and may be seen as a strand of discursive psychology (Harré & Gillette, 1994). Rooted in Wittgenstein's philosophy and Vygotzky's theory of learning, the commognitive theory's basic claim is that thinking can be defined as an intra-personal type of communication not qualitatively different from interpersonal communication. Accordingly, Sfard (2008) coined the term commognition (composed of communication and cognition) to stress the inseparability of these two phenomena. The focus on communication is pivotal for the present study's goal as it removes the dichotomy between cognition and emotion. When looking at human communication, it becomes obvious that in real life, this activity has emotional and cognitive aspects to it that are mostly inseparable.

Based on the commognitive framework, Ben-Yehuda and her colleagues have used a *mathematical interview* devised by Ben-Yehuda (2003) to study the discourse of two low-achieving high-school girls, who both had a personal history of educational impoverishment. They showed how one of the girls, Talli, who identified herself as good in mathematics, and was identified by the teacher as having good potential, was actually performing calculations in a strictly *ritual* and *syntactic* way. That is, she was relying solely on externally given rules of replacing digits with other digits, without showing any signs of *objectification*. Objectification is talking about mathematical objects as entities in the world, for instance, talking about "the number four" as existing on its own, instead of saying "four is the number-word that I end up with when counting one, two, three, four". Sfard and Lavie (2005) have postulated that objectification is a necessary step towards developing a more sophisticated mathematical discourse. For instance, a child has to objectify the number four before she can talk about "four plus four *is* eight". The second girl in Ben-Yehuda and her colleagues' study, Mira, identified herself and was identified by her teacher as very weak. Her mathematical discourse, however, was much more objectified than that of Talli, and yet she restrained herself from relying on her own object-mediated routines of calculation because she deemed them as inappropriate for her age (for instance, she was embarrassed about counting on her fingers). The authors thus postulated that the reasons she was not able to progress satisfactorily in mathematics had to do with her identity and with the norms she was attempting to follow of how mathematics should be done.

2.2.1 Commognitive tools for studying affective phenomena in learning

The most common concept used by socio-cultural theories to study affective phenomena and the place of the individual in the learning community is *identity*. Students' identity has been the focus of an ever-growing amount of research in educational settings in general (e.g. Gee, 2000–2001; Leander, 2002; Wortham, 2006), and in mathematical learning in particular (Andersson, 2011; Bishop, 2012; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Ingram, 2011; Lange, 2009; Nasir, 2002; Solomon, 2007). However, the exact definition of what identity is has been a source of much debate (Brubaker & Cooper, 2000). In an effort to overcome the blurriness of this term Sfard and Prusak (2005) defined identity as a "set of stories". More specifically, they defined it as "a […] collections of stories about persons […] that are *reifying, endorsable,* and *significant*" (p. 16). The reifying quality of an identity story is achieved when talking about what people *are* as opposed to what people *do*. Endorsability was operationalized by Sfard and Prusak by "if the identity-builder, when asked, would say that it faithfully reflects the state of affairs in the world". The significance of the story would be determined, according to this definition, "if any change in (the narrative) is likely to affect the story teller's feelings about the identified person" (p. 17).

The above definition of identity was endorsed by many researchers in the domain of mathematics education in recent years (Andersson, 2011; Graven, 2011; Lange, 2009; Ingram, 2011), yet it is mainly helpful for operationalizing identity when people are explicitly interrogated about it (such as in interviews). It is less useful for the study of identities as they are constructed through classroom activity. One of the aims of the larger project of which this study was part was to create a set of operational criteria for determining what could count as an identity narrative from the spontaneous discourse of students in class. Therefore, I moved from looking at identities as end products to analyzing *identifying* activity, or the activity of authoring one's own and other's identities (Heyd-Metzuyanim & Sfard, 2012; Wood & Kalinec, 2012).

Identifying is somewhat parallel to *positioning* (Davies & Harré, 1990; Harré & van Langenhove, 1999) that has been a very popular concept in recent years for describing social interactions in class (Anderson, 2009; Black, 2004b; Evans et al., 2006). However, for the present research purposes, "positioning", as useful as it may be, has some limitations, one of which is its fleeting nature. At every moment, the student may position herself or be positioned by others in different ways. Similar to positioning theory, the communicational framework conceptualizes identity as social and situated (as all discursive moves are); yet in this study I am still looking for the relatively *stable* narratives of students about themselves that are carried over from one learning situation to the other.

One of the drawbacks of using the identity concept is that it tends to become objectified. In other words, a person is said to "have" a certain identity, without explicating who exactly has authored this identity (the person about herself or others about her?). The concept of *identifying* attempts to cope with this blurriness by stating explicitly *who* identifies *whom* and to *which* audience. Borrowing from Sfard and Prusak (2005), I explicate this information by characterizing each identifying utterance with the threesome $_{\text{Author}}\text{Subject}_{\text{Audience}}$. Utterances of the type $_{A}A_{B}$ are 1st Person identifying made by A about herself, when talking to B. Correspondingly, utterances of type $_{B}A_{C}$ are 3rd P identifying and $_{B}A_{A}$ are 2nd P identifying. Making these threesomes explicit may provide important insights into how different stories about the same person (such as 1st P and 3rd P stories) are related to each other and how they influence one another.

Sfard and Prusak (2005) made another useful distinction between two types of identity stories. The first is *current* identities, which are stories about the current state of affairs (such as "I'm a lousy mathematician"). The second type is *designated* identities, which are stories about how things are expected or supposed to be ("I want to be in advanced mathematics"). They showed that students with different designated identities (for instance those who see mathematical education as part of their being cultural people in contrast to those who see mathematics as a tool for entering prestigious academic tracks) study mathematics differently. They thus postulated that closing the gap between current and designated identity is actually a primary motive for learning. In the present study, I shall use the term *designated identifying* to characterize talk about how things *should* be done or how people are *expected* to participate in certain activities.

2.2.2 Extracting identifying activity from talk

While every person has a multitude of stories told about him, extracting these stories from the every-day interaction between people proves to be quite a complex task. The first step I take towards capturing identifying activity is by dividing students' discourse (both verbal and non-verbal) into two main categories: mathematizing and subjectifying. *Mathematizing* is defined as communicating (whether orally or in written form) about mathematical objects, using mathematical words and mathematical signs (for instance "2+2=4" or "an isosceles triangle has two equal angles"). *Subjectifying*, on the other hand, is defined as *communicating about the participants of the discourse* (such as "I don't know how to solve this problem" or pointing at a student and nodding in approval).

The clearest candidates for observing identifying activity are verbal subjectifications that talk about a person explicitly (and thus are called *direct*). Those can be divided into three levels. The first is the *specific* level, which are statements pertaining to a participant's actions in a specific context. For instance: "I didn't get this", or "I think I know the answer". The second level consists of *general* evaluations of one's participation in the discourse. For example "I **never** succeed with these kind of exercises", or "**She's always been** irritated by fractions". The most general and stable level is that of attributing *properties* or assigning *membership* to a person. Such are statements like "**She's** a straight A math student", "**I'm** bad with fractions", or "**He has** a learning disability". This highest level is identifying by definition. However, scrutinizing general participation evaluations and even specific subjectifications can give a clear picture of a student's identity, provided these statements are *recurrent* and *consistent*.

2.2.3 Implicit and indirect identifying

If all students and teachers would talk in a straightforward manner about themselves and about others, we would have little problem in re-constructing their identities from the above three levels of direct subjectifying. However, as social and cultural norms significantly constrain what is considered to be appropriate to be spoken out loud, it is necessary to devise ways to consistently and operationally interpret identity narratives from indirect clues. I use the term *implicit* and *indirect* identifying, to denote utterances that, though not talking specifically about a person, still convey an important message about him/her. Indirect identifying utterances are those that, while directly making a statement about person B, also indirectly state something about person A. Implicit identifying utterances are those that implicitly state a message X about a person, while explicitly stating the message Y. For instance, suppose Bob and Tracy are faced with a problem and Tracy says to Bob: "you should solve this problem". Tracy can be interpreted to *implicitly* be stating about Bob: "Bob is competent in such problem solving" while *indirectly* stating about herself "I am *not* competent in such problem solving".

2.2.4 Interactional routines

As any discursive object, identities are constructed through social interaction (McCall & Simmons, 1966). These social interactions are patterned activities, highly constrained by social and cultural norms (Goffman, 1967/2005). The constraints, or meta-rules (that is discourse about discourse) that describe a repetitive discursive action have been defined by Sfard (2008) as *routines*. Such meta-rules do not have to be articulated by the participants of the discourse. On the contrary, they remain often implicit and are only articulated by external observers who watch the participants' actions carefully for repeated discursive actions.

Regarding classroom life, interactional patterns have been studied extensively, both in relation to general teaching–learning interactions (Cazden, 2001; Lefstein, 2008; Mehan, 1979) and in relation to mathematical learning in particular (Cobb, Yackel, & Wood, 1992; Voigt, 1994). Such sequences of interaction do not necessarily belong to one particular discourse (such as the mathematical one), but to what is often referred to as the social norms of classroom practices. Other implicit rules that govern classroom teaching–learning interactions refer to what is accepted as a mathematical argument or mathematical ways of solving problems. Those have been termed as *socio-mathematical norms* (Cobb et al., 1992) or simply *meta-discursive rules* of the mathematical discourse (Sfard, 2007).

In the present study I have come upon the need to articulate routines that are somewhat distinct from the categories of social norms, socio-mathematical norms and mathematical routines. Those routines belong to an interim domain that partially includes all of the above categories, but is not completely congruent with any one of them. They can best be described as meta-discursive rules for determining the *subjectifying* that is appropriate for certain *mathematizing* actions. For instance, answering the question "how much is 8 times 2?" with "I know! It's 16!", or with "uh.. I guess 14? No…" have both very different mathematical messages as well as different subjectifying messages. The first constitutes "correct", or generally endorsed mathematical narratives, together with communicating self-assurance. The second constitutes "incorrect" mathematizing together with an appropriately hesitant hue. These examples are pretty straightforward instances of normative interactional routines. However, such interactional routines may be much more complex and implicit. In such cases, they are best identified by their *violation* (Voigt, 1994). Such a violation is demonstrated in the following excerpt. In this classroom episode, I (the teacher) introduced a new subject (that of "number formulas" or algebraic expressions), to a group of four low-achieving mathematics students, including Dana, who was at the focus of this case study. Though all participants had very little experience with the words “number formula”, Dana’s answer to my question was somewhat exceptional:

Excerpt 1—Dana’s violation of an interactional routine

Turn	Speaker	What is said
1	Teacher	Today we start talking about number formulas[1]. Who knows what a number formula is?
2	Hadassa	The formula of the number, like uh..two eh two and one like?
3	Teacher	Umm not quite. Someone else has an idea of what is a number formula?
...		The students do not seem to comprehend what the teacher is looking for so the teacher tries again:
18	Teacher	For instance, uh..with formulas, yeah? Have you seen once a formula of something?
20	Nadav	Yeah, of an experiment
23	Hadassa	Ah! Yes I saw such an experiment, uh uh, P plus P and A[2]
24	Nadav	In “the eights”[3]
25	Teacher	Good. For such formulas we need letters, OK?
26	**Dana**	**P plus P, I didn’t get that stuff with the E.. For instance E plus seven. I didn’t get that stuff**[4].

Though obviously, no student in this group had an idea of what I (the teacher) was talking about when saying “a number formula”, all of them provided answers that somehow conformed to the interactional routine appropriate for this scenario. This interactional routine could be described as follows:

> **Initiation**: The teacher asks a question about some new term, in this case “number formulas”.

In reaction to this initiative, there may be several plausible reactions:

> **Response 1**: the students answer their best guess of what this term means, acknowledging (perhaps non-verbally) that they are only guessing.
> **Response 2**: the students have some experience with the new term, and therefore use it according to the conventional rules. Example: “Yeah, a number formula is something like 2b or x+4”.
> **Response 3**: The student has some experience with the new term, but acknowledges she yet has to understand it better. She thus uses the term appropriately, but asks for some clarification about it. For instance: “Number formulas are those expressions with Xs, but I don’t understand when you use Xs and when you use Ys.”

Hadassa’s answer [2]: “The formula of the number, like uh..two eh two and one like?” with its hesitant hue, provides a good example for a response of the first type, where the student attempts to use a new word, while acknowledging her inexperience with it. Nadav’s answer [20] provides an example of a type 2 response, where the student is able to use the new word in a normative way (though not in the exact mathematical sense). In contrast to

[1] Heb: “Tavniot mispar”; “Number formula” is the Hebrew term that refers to algebraic expressions. I chose here to use the literary form because the students refer specifically to the word “formula”.
[2] All the students articulated the English pronunciation of the capitalized letters
[3] “The eights” was a popular Israeli T.V. series in that year.
[4] Heb: “Keta”, Lit: a section. The word “Keta” is very common in Hebrew slang for denoting some piece of information. The closest translation would be something between “a thing” and “stuff”.

these responses, Dana's answer: "P+P, I didn't get the thing with the E, for instance, E=7" was deviating from these normative interactional routines by presenting a curious mix of the above options. She used the *subjectifying form* of Response 3 (expressing some confidence about partial acquaintance with the new term while requesting elaboration on a specific point) while her mathematizing fitted Response 1 (of having no experience with the mathematical term and how it should be used).

This example shows that interactional routines in teaching–learning situations may be very complex to describe, not only because they are never explicitly stated, but because they combine both subjectifying messages (such as "I know what I'm talking about", or "I'm only guessing") with mathematical or other content-specific rules. Thus, unless we know how expressions such as "E equals 7" are used in mathematical discourse at schools, or "I didn't get the thing with the …" are used in the every-day discourse of 7th graders, we would not be able to see the oddness of Dana's utterance.

Deviations from normative interactional routines often lead to emotional reactions, such as amusement or feeling that something is odd. Moreover, they may elicit strong identifying statements about the person who deviated from the interactional routine. In the above example, for instance, Dana's deviation from the interactional routine (as I had expected it to proceed) led to my identification of Dana as "clueless" and funny (though of course this identity was held privately, as a $_{\text{Teacher}}\text{Dana}_{\text{Teacher}}$ story, evidenced only in my research diary). It is for this reason that following the interactional routines of participants in learning–teaching interactions may tell us much about what led to the construction of certain identity narratives.

3 The case of Dana

Now that the conceptual tools for studying the relationship between emotions, social interactions, and mathematical failure have been defined, the question of the present study may be formulated in operational terms. It is:

What are the mechanisms of interaction between identifying and mathematizing that may be responsible for repeated failure in mathematical learning? In particular, how do the different stories about a student (such as those told by the teacher and those told by the student) affect the opportunities that the student is offered to develop her mathematical discourse?

In order to answer this question, I chose to focus on the case of Dana, who was the lowest achieving student in the larger study I was conducting (Heyd-Metzuyanim, 2012; Heyd-Metzuyanim & Sfard, 2012). This study included 12 7th graders who participated in a 5-month course in an out-of-school learning center. The students were divided to three groups of four according to their prior achievements. Dana was part of the low-achieving group, and was reported (by her school teacher) to have had failing grades from the very start of 7th grade. Dana's mother, who was interviewed at the end of the course, told of long-lasting difficulties Dana had had with mathematics since the very start of her schooling history. According to the mother and the counselor at school, Dana had been diagnosed at 4th grade as learning disabled and as having ADD[5].

In addition to the lessons (referred to from now on as the Course), each participant in the study underwent two interviews, at the beginning and at the end of the course. The first interview included a very short acquaintance part, and a mathematical assessment, designed after Ben-Yehuda's (2003) mathematical interview which had been developed for the same age group as that of the current participants. The second (final) interview included two parts. The

[5] For technical reasons, I did not get straight access to Dana's psychological diagnosis.

first was an interview aimed at eliciting identity narratives from the students, aided by a movie which described the anxiety of an 8th grader who for the first time in his life encounters difficulties with mathematics. The second part was a repeated mathematical interview, with some tasks replicating the first interview, and others taken from the Course's curriculum.

Dana's case was perplexing because in spite of her visible effort and her declared wishes to advance in mathematics, and despite the intensive instruction she had received in my Course, in school lessons and during a weekly private tutoring hour from her school teacher, she did not seem to individualize any of the new mathematical skills taught at that year. To understand what was hindering Dana from any progress, I shall first analyze her mathematical discourse at the beginning of the course, extracting from it her mathematical tool-set—the set of mathematical words and routines she had not yet mastered. Next, I shall move to examine closely her forms of participation in class and the typical interactions she had with me, her teacher. I shall show that the interactional routines Dana and I were following were very different, leading to non-productive mathematical interactions and a co-construction of Dana's identity of failure.

3.1 The first mathematical interaction between Dana and me

Looking back, I recall identifying Dana as extremely low achieving in mathematics (or "clueless" in colloquial terms) from a very early stage of the first interview. How was my 3rd P identity of Dana constructed so early? It should be noted that, though I had some idea of Dana's low achievements at school from preliminary information given to me by Dana herself and by her mother, I did not know much about Dana and her specific mathematical skills before this interview had started. And yet, it did not take me more than a few minutes to identify Dana as significantly deviating from the standard mathematical performance of a 7th-grade student. In what follows, I shall examine this swift identifying process from the lens of interactional routines that were successively violated by Dana in the interview.

The interview started by my explicit declaration of the interactional routine that I was designating for the interview—that of hearing the student think out loud:

> 198. Teacher: … I will ask you to think out loud. Whatever you do, (what you) calculate inside you head, say out loud. OK?
>
> After this introduction, Dana read the first task which was:
>
> write the appropriate numeral (translate from words to symbols) Sixteen million, six hundred and five thousands, and nine
>
> Then, she immediately stated:
>
> 199. Dana: Oh, I got it, sixteen. That's like they do in 4th grade, write the number.

The beginning of this interaction does not seem conspicuous. Though Dana did not immediately come up with the answer, she did express a 1st P evaluation of understanding what the question was about. However, after a few hesitations, where Dana wrote and erased some figures, a deviation from the normative routine occurred:

> 207. Dana: What, this alone, this alone and this alone? (pointing to the comma separated number-words)
> 208. Teacher: It's one number. Sixteen million, six hundred and five thousand and nine.

Dana's question, in addition to her relatively long-lasting indecision about the right answer, was violating the interactional routine of "presenting a question, responding with the right answer" that would identify Dana as competent in solving these types of tasks. However, this deviation was not particularly exceptional in my view. Though the task was indeed taken from the curriculum of 3rd or 4th grade, several students in the study erred in it, and even more of them had some initial hesitations.

After a few more trials and hesitations, another interaction occurred, this time deviating more significantly from the expected interactional routine.

239 Dana: So wait, how do I do (it)? I do comma, and then… zero zero zero, and zero zero zero zero? Or I do, sixteen, zero zero zero, comma, zero zero zero zero?
240 T: You have to insert the whole number at once (*making a squeezing gesture*)
241 Dana: Oh! So no commas (*Nodding quickly, in a relieved tone of voice*)
242 T: You know what we'll do? Let's go on to the next question, and if it will ring a bell, then maybe we come back to this one. OK?

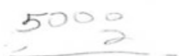
Clearly, at the end of this exchange, I had identified Dana as unable to perform the task, which is why I suggested going on to the next question. What was Dana doing that was violating the interactional routine in such a way that I gave up on trying to elicit a correct answer from her? The first answer can be found in the mathematical content of Dana's words, and especially in the forms of mediation she was using. While Dana was mediating the task in a *syntactic* form, that is, she was mainly referring to digits and "commas" [239, 241], I was talking about the number in an *objectified* way ("you have to *insert the whole number* at once"). Moreover, having objectified numbers in the range of a million long ago, I had difficulty relating to them in any other way. Thus, to Dana's initial question ([199]) I replied with "It's one number. Sixteen million, six hundred…". Obviously, this answer did not make much difference to Dana, as it was simply repeating what was stated on paper.

Thus, the first hurdle to holding a successful interactional routine between Dana and me was based in the very different mathematical skills each of us had. And yet, this difference was, as mentioned before, not particularly unique to this interview. In fact, in any novice–expert interaction such a difference is the norm rather than the exception. The more significant deviation from the interactional routine was in the identifying messages uttered throughout it. While the norm would be that a student who fails to communicate in the same manner as the teacher would somehow acknowledge it (for instance, by communicating confusion), Dana was stating that she understood me ["oh! So no commas", 241], while in my eyes she failed to do so completely.

And yet, my 3rd P mathematical identity of Dana as "clueless" was not completely formed yet, in part because the task of writing a number in the range of a million was not considered by me as such a necessary part of the student's mathematical skills at this stage, and in part because Dana had succeeded (with some assistance) in the next task (reading out loud the number 20,304). The third task, however, was much more detrimental to my formation of Dana's identity as a capable mathematical student. In it Dana was requested to solve the exercise 5,000–2.

Excerpt 1—Dana's mathematizing in the 5,000–2 task

Legend: (.) short pause; (….) long pause (dots signal number of seconds); :: prolonging of a syllable;

Turn	Speaker	What is said (*What is done*)
273–4	Dana	(……..)*writes*  *then stops and thinks*(..)
275	Dana	Four hundred (.) Four hundred forty:: (.) seven. No. Forty eight.
276	Interviewer	Four hundred forty eight?
277	Dana	(*Shaking her head quickly, signaling "I mixed it up"*)Four **thousand** forty eight

The first, very prominent violation of the normative interactional routine here is of course in the mathematical content of Dana's answer, that 5,000–2=448. However, this violation, common to any student who errs in a mathematical routine, is followed by adherence to the common routine in which the interviewer asks a question "just to make sure", signaling that the answer should be corrected. Indeed, Dana adheres to this interactional routine by correcting her answer. And yet, the deviance from the routine is maintained by the remoteness of Dana's correction, 4,048, from any accepted mathematical narrative.

After a few probes, in which Dana changed her answer to 4,058, wrote it down and seemed satisfied with it, I attempted to challenge this answer by asking:

290	Interviewer	If we add two what will we get here? (*points to 58 in 4,058*)
291	Dana	Two plus fifty eight
..		
294	Dana	No, we'll get more, we'll get uh. For instance (*Looks at the exercise, then up at Teacher*) five thousand and one.. so minus one[6]
295	Teacher	Why will we get five thousand and one?
296	Dana	Because..if you do, for instance, you have five (*raises five fingers*) OK?
297	Teacher	Yes
298	Dana	So, so you have to take off two (*bends down two fingers*) (.) three ^thousand^! (*Expression signaling "I got it!")*
299	Teacher	(*mumbling hesitantly*) OK
300	Dana	Three million! (*The enthusiastic tone, ends with a desperate hue*)

Again, this interaction started off with adherence to normative mathematical routines (Dana responding with "we get two plus fifty eight" [291]), but quickly swayed from them when Dana claimed that adding 2 to 4,058 would result in 5,001 and in her conclusion "so minus one" [294]. Both these claims seem non-comprehensible according to normative mathematical rules. And yet, the mathematical content of Dana's words did not deter the interaction from going on, with me asking Dana to clarify her reasoning [295]. Again, Dana started by adhering to an interactional routine where a participant explains her mathematical reasoning using concrete mediation (In this case, Dana used her fingers to explain why 5–2= 3 [298]). But immediately after that, she deviated from it by jumping with a new idea: "three thousand!" [298]. When this answer was met with silent disapproval, she jumped to "three million!" [300]. Both last exclamations, and especially the "three thousand!", were communicating non-verbally the message "I got it!" while my own evaluation was "this answer is completely wrong". Therefore, once again, Dana was identified by me as "clueless", yet this time this identification was much stronger because it involved a task that I had expected to be successfully performed by almost any 7th grader.

The most illustrative example of Dana's deviation from normative mathematical interaction comes from her solution to a real-life word problem. Here, Dana's odd 1st P evaluations were not limited to non-verbal communication, but authored in a direct verbal manner, as underlined in the text. The problem presented to Dana was:

> 4 Shirts cost 200 NIS. 6 Pairs of pants cost 300 NIS. Rina bought 2 shirts, and 2 pairs of pants. How much did she pay for the whole purchase?

[6] Heb: "Az pakhot ekhad" Lit: so subtract one. "Pakhot" is the word for the subtraction sign, and is not used in this context as "minus" which might signify the number (−1).

Excerpt 2—Dana solving the "shirts and pants" problem[7]

Turn	Speaker	What is said (*What is done*)
577–580	Dana	(*After reading the question out loud*) ..Um ..fifty, five hundred, another five hundred ..
581	Teacher	You said fifty and then five hundred. I didn't understand that well.
582	Dana	All right, this turns five hundred, right? (*points to the written problem*)
583	Teacher	OK
584	Dana	And ^twice more^, five hundred (*tone meaning "what's the problem?"*)
		So it turns..a hundred. No, (it) turns thousand five hundred. Oysh, wait. (*Taps her pencil on the table, leans forehead against hand, smiling shyly*) Uh.. it turns… >how much is five hundred plus five hundred?< (*frowns*)
Teacher asks Dana to read the question again, and starts reading it for her: "Four shirts cost two hundred shekel"		
588–9	Dana	Oh! Four Shirts! (*Interviewer goes on reading the whole question*)
590	Dana	Ahh! I got it! Whoa! How can I do four in- Oh! Four minus six, (.hhh) (*straightens up enthusiastically*) (you) do (.) this (*points to 200*) and this (*points to 300*), minus this (4) and minus this (6) and minus this (2) right? All the numbers (you) do minus.
591	Teacher	(*Chuckles*) OK
592	Dana	(*Raises two hands, signing apology*) that's what I was taught, I don't know any other (way)
593	Teacher	No, all right
594–596	Dana	Um.. it's a wonder that I still remember this, but cool. Uh, OK Four minus six …. (*Straightens up, serious expression, then bends down, rolls her eyes and smiles*)^Oh^[8] What's with me today?(*smiles, covers her mouth*). I don't remember anything (*looks down, frowns*) Um ………………
597–601 *Dana clarifies that she's "stuck" on 4–6. Teacher starts suggesting "maybe another way" but Dana intervenes*		
602–604	Dana	(*After looking at her fingers*) two (Teacher: OK) So um..here it's together two (*writes "2"*) um.. five hundred (*writes 500*) .. four (*writes "4"*). Now, I do it all together, right?
605	Teacher	OK
606–608	Dana	Five hundred, plus two, plus four. So five hundred.. s- and six, five hundred and six. (Teacher: OK) It's right, right?
609	Teacher	^Not^ exactly, but-
609b	Dana	*Drops her pencil, grasps her head with her hands in a disappointed expression*

Again, the peculiarity of Dana's performance here is not so much in her mathematizing, though that might seem on a first look to be associative guessing at best. But before looking at Dana's deviations from the normative interactional routines, let me claim that on a close look, one can see some of her mathematizing is, in fact, following generally endorsed mathematical rules. For instance, Dana's first claim, that the answer should be "twice five hundred" can be explained as a result of a reading comprehension error that led her to think of the problem as "**A** shirt costs 200 shekels and **a** pair of pants cost 300 shekels". From this point on, however, Dana follows very different rules for mathematical problem solving from those of competent participants. Thus, instead of relating the expression "4 shirts" as an indication to *multiply* "4 X price of shirt" she treats the digits 4, 6, and 2 as signaling she should subtract or add them in some sort of orderly way. This syntactic treatment of a word problem may seem very strange, unless one takes into account that multiplication routines

[7] Legend: ^high pitch^; .hhh: audible inhale; >accelerated talk<

[8] Heb: "Dai", Lit: enough, stop it

were very rare and ill performed in Dana's discourse (as could be seen in other parts of the mathematical assessment). Without having mastered the multiplication routine (not only the *how* but also the *when* of this routine; Sfard, 2008), Dana was constrained to making the best of the routines of which she had mastery, which were addition and subtraction.

Thus, analyzing the mathematical content of Dana's words, one can see that though it deviated strongly from normative mathematical routines, still some order could be found in it. This order completely escaped my eyes at the time of the interview, probably because I was distracted by Dana's violation of the identifying aspects of the interactional routine, the most prominent of them seen in her exclamation "it's a wonder I still remember this, but cool" [594]. Here, not only was Dana directly subjectifying herself as a competent participant in the interaction (a narrative that I was obviously opposing), she was also referring to the whole process as something that should be remembered, a sort of mysterious routine practiced in long-forgotten 4th-grade lesson. In contrast, I was identifying the task as a very simple arithmetical problem, encountered in everyday life. We were differing thus not only on the mathematical routines that should be practiced on such an occasion, but also on how the task was situated in the world, and who was expected to be able to perform it.

The rest of the first mathematical assessment was not very different from the above few examples. On the whole it seemed that by the time Dana had entered the Course, she had roughly the mathematical skills expected of children at around 2nd grade (see Appendix, Table 4 for a summary of her performance on the first assessment). And yet, Dana was continually expected to participate in mathematical discourses that were beyond her skills. Therefore, she had developed *alternative* routines so that she could adhere to this designated identity. Those routines could be roughly described as:

a. In any mathematical task, or exercise, look at the digits and try to find some way to add them, subtract them, or move them from place to place in some orderly manner.
b. As the above strategy often proves to be unreliable, place whatever mathematical idea you have for external evaluation by the teacher, so you may be steered into acting correctly before you lose your way. Throw out your associative ideas as quickly as possible so one of them may be caught by the teacher and declared as right.

Looking at the mathematical skills Dana had mastered, in relation to those she was expected to exhibit, it becomes clear that Dana did not, in fact, have much other choice. As long as she wished to identify herself, even partially, as mathematically competent, she had to use whatever means were available for her, even if those were constrained to simply expressing her understanding. However, identifying herself as a competent participant proved to be counter-productive as it repeatedly violated the subjectifying aspects of the normative interactional routines, leading to my identification of her as "clueless", even somewhat "hopeless" in mathematics. This was most prominently expressed in the following diary entry I made shortly after the first interview with Dana:

> It's unbelievable … More shocking than her (Dana's) lack of knowledge, her behaviors in calculations are amazing. She just throws millions of guesses without any connection between them … The mess that she has in her head with math is unbelievable. When to do addition, when subtraction, how to do it, everything is in a big chaos. (My research notes, 10/2/08)

In the next section, I shall examine how this 3rd P identity I had formed of Dana from the very beginning of the Course hindered us from having productive mathematical interactions that would enable Dana to advance in mathematics.

3.2 Learning–teaching interactions between Dana and me

Having both co-constructed in the first interview an interactional routine in which Dana throws out guesses and I evaluate them as right or wrong, there was not much chance for the formation of explorative discourse between Dana and me. My identification of her as "clueless" led me to believe that not much could be gained from asking Dana to express any mathematically rational arguments.

And yet, some rare chances for Dana to provide explanations of her thinking did occur, for instance in the first lesson, where I presented the students in Dana's group (Dana, Hadassa, Hagit, and Nadav) an unconventional task, aimed at eliciting a mathematical discussion about generalizing arithmetical calculations. The task was named "the ladders problem" and was presented as follows (see Fig. 1):

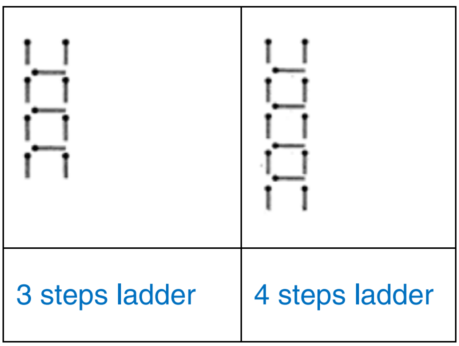
> Ladders are built using matches. … With 11 matches, it is possible to build a ladder with 3 steps. With 14 matches, it is possible to build a ladder with 4 steps.
>
> The first, simplest question was:
>
> How many matches would be included in a 2 steps ladder?

Nadav and Hagit claimed (correctly) that the answer is 8, while Dana and Hadassa insisted it was 6. I invited each of the participants to explain his/her solution and try to convince the others. At first, Hagit explained why the answer was 8:

> 90–95 Hagit: If you build a regular ladder.. then you need here (*pointing to the lowest part of the ladder*) to have a supporter… Right? So here you have a supporter.
> 96 Dana: My darling, it's six, it's six!
> 97 Teacher: No, listen to her, listen to her (*with an impatient tone*)

Remaining unconvinced, Dana volunteered to offer her own explanation:

Fig. 1 The ladders in the worksheet



Excerpt 3—Dana's explanation in the "ladders problem"
Legend: >quick speech<, [overlapping speech]

Turn	Speaker	What is said (*what is done*)
106	Dana	Imagine you have, >wait wait wait<(*stands up*)
…		
108	Teacher	>wait, wait, wait<, Dana, sit down [If you want to show them draw on a paper]
109	Dana	[it's like this sort of] *(sits down)* (*Using a "teacher voice"*) imagine, en-vision[9] (*waves up her forefinger*), envision if you will[10], (*whispers down to Hadassa*)>I learned that in—<
110	Teacher	(*In a slightly impatient tone*) Mm
111	Hadassa	*(Looks from Dana to Teacher, chuckles slightly)*
…		*(Dana looks for a few seconds over at Hadassa's sheet, trying to figure out something)*
115–116	Dana	^OK^ en-vision that you have (*whispering, pointing to her paper*) one, two, three, four, OK? Let's say. Sorry (*mumbles something, looks at her pencil case, Hadassa gives her an eraser*)
117	Dana	Thank you very much sweetheart. Wait, oysh (*erases, nods towards Hadassa in a dramatically thankful gesture*)
118	Hadassa	(*chuckles*, *looks at the teacher*) As usual
119	Teacher	Umm.. OK
120	Dana	Envision you have (*points to her paper, speaks in a "teacher" voice*) one, two, three (*now in her own voice*) four, five six and seven (*Looks at the teacher*)
121	Teacher	Mm
122	Dana	Oops! (*laughs*) Wait. (*counts the lines on her paper*)>one two three four five six<^six^ (*smiles and claps her hands, signaling success*)
123	Hadassa	OK I want to say something
124	Dana	No! It's actually seven!

The heart of the mathematical dispute in this episode was a very simple matter. Whereas Nadav, Hagit, and I realized the signifier "two steps ladder" as depicted in Fig. 2 (B), Dana and Haddassa realized it as depicted in Fig. 2 (A). And yet the discussion, as led by Dana, was far from following the normative interactional rules of mathematical discussions. Table 1 summarizes all these deviations.

As demonstrated in Table 1, the deviations Dana made in this episode from normative interactional routines abound. The reactions to these deviations could be seen in the emotional expressions of the participants, for instance, in my explicit request from Dana to sit down [108], and in my un-amused responses to her drama act [110, 119]. But I was not the only one signaling some interactional routines had been violated. Hadassa, who was sitting next to Dana, was communicating her embarrassment, even apology, for Dana's drama act ([111],[118]). Dana, however, was undeterred by these minimal signs of disapproval. On the contrary, she went on with her drama act apparently unaware of how it was identifying her in my eyes as incompetent to participate in such a discussion.

This 3rd P identification was detrimental for Dana's future interactions with me. From that point on, I hardly ever engaged her with mathematical thinking problems. Instead, I concentrated on practicing and drilling the material that was taught at school, hoping that by simplified, direct instructions of how the mathematical routines should be performed, I would help her to narrow the immense gap between her current skills and those expected of her at school.

[9] Heb: "Tedamyenu lachem, Da-mu lachem"; Dana is changing slightly the verb of "imagine" to be in the correct form, which is only used in literary Hebrew, thus exaggerating a "high" form of speech.
[10] Heb: "Damu-na"—a literary form of saying "imagine please", hardly ever used in today's spoken Hebrew.

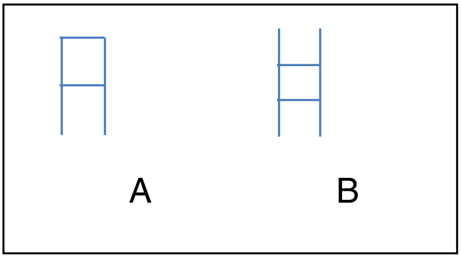


Fig. 2 Two realizations of a two-steps ladder

The following episode, taken from the second half of the first lesson, exemplifies a very typical episode of our teaching–learning interactions from that point on. In it, I presented the students with a worksheet containing exercises of adding positive and negative numbers. I explained the method with which the students were supposed to solve these problems that involved using right arrows to signal positive numbers and left arrows to signal negative numbers. I then handed the students a worksheet with number lines (with zero at the middle) on which they were supposed to practice "going left" and "going right" for adding negative and positive numbers. For instance, the calculation (+4)+(−1) would be calculated by moving from zero four arrows to the right (landing on +4) and then one arrow to the left. When I first explained this method to the group, Dana excused herself out of the class for a few minutes. Therefore, she received an individual explanation when she came back:

Excerpt 4—Teacher explains how to calculate addition of negative numbers

6.	Teacher	Do you know to do addition of negative numbers? Say negative three plus negative one?
7.	Dana	(*Squints sideways*, *with a hesitant tone*) I guess so
8.	Teacher	Come, I'll remind you of the method. That's the method I start with, with the arrows (*points on the paper*), OK? (*Dana coughs, nods*)
		Arrows. Left, is the minuses (*Dana nods*)
		I start with the zero
9.	Dana	(*points to the worksheet with the empty number lines*) Uh, we get such a paper?
10.	Teacher	^Yeah^
		(*Dana nods, then starts looking around, gazing at Nadav's work*)
		I start with the zero and then I have three. My exercise is minus three plus minus one, OK?
11.	Dana	(*Squints for a second at the teacher's pointing, then looks back at Nadav's work*) OK
12	Teacher	And I start at the zero and go three steps to the left. 'Cause it's a minus.
13	Dana	*Nods, her gaze wandering around the classroom*
14	Teacher	One, two, three. OK? (*Dana doesn't answer, her gaze wandering*)
		Now I add another arrow
		(*Dana squints over to the teacher's paper for a second)*
		That is also in (that) direction *'cause* it is minus one
15.	Dana	OK (*looks around again*)
16.	Teacher	I got together to minus four
17.	Dana	*Nods slightly, gaze wandering*
	The teacher gives another short example of (−3)+(+1), then hands Dana a worksheet.	

Table 1 Deviations from normative interactional routines in the "ladders problem"

	Normative	Dana's performance
1. Mathematical routines	Counting the matches correctly up to eight. Realizing ladder as in Fig. 2 (B).	Stating the number of matches is 6; counting incorrectly up to 7 [120]; realizing the ladder as in Fig. 2 (A).
2. Subjectifying messages	Acting as an equal discussant with the other group members and as a rational student in general.	Acts as superior to Hagit ["my darling, it's six", 96], and to Hadassa ["thank you sweetheart"]. Acts as a funny teacher.
	Stating calmly and rationally her opinion (as Hagit did in [90–95]).	Acts excited; stands up; uses an artificial "teacher" or mothering voice.
	Upon making a generally unacceptable mathematical statement, express hesitancy.	Expresses certainty and evaluates herself as being right even though the mathematical statement is unacceptable [122].

At a first glance, it seems that both Dana and I were adhering in this episode to a well-established interactional routine between a student and a teacher. I was explaining how to do the exercises, while Dana was listening and nodding. However, on a closer look, there was practically no effective mathematical interaction here. Not only was I (the teacher) doing all the mathematizing, but Dana was not listening at all, as could be seen in her wandering gazes that were almost never focused on my pointing. At the time, I was completely unaware of Dana's inattentiveness, perhaps because Dana managed to trick me into believing she was listening, by superficially adhering to the expected (non-verbal) identifying part of the interactional routine. Not surprisingly, therefore, Dana was not able to carry out my explained routine on her own. The minute she laid her eyes on the first exercise in her worksheet, she exclaimed "this is difficult!", and requested my explanations again.

When Dana was attentive, it was mainly for eliciting from me "how to do" directions. See, for instance, her questions directed at me only a few minutes after the above interaction:

76.–77.	Dana	(*writes something on her worksheet)* oh, oh, listen (*walks up to the teacher)* **What do I do** in this situation, that I have this plus this, like, here (*points to (+4)+(−1))* I write the biggest?
78.	Teacher	First of all this is excellent (*marks a V on a former exercise that Dana had solved correctly*). What are you asking? About plus four and minus three?
79.	Dana	Like, **what do I do**, the closest to zero, for instance plus three, minus three?
80.	Teacher	You start, look. **You do**.. uh with the arrows**.** Plus four is (going) right, right? (*points with thumb to the right)*, so one, two- this is zero (*points to the zero on the number line*) so one, two, three, four. I arrived at four. Now what do I have to add? Minus one, which is one arrow (points to the left), one arrow to the left. OK? I arrived at the three.
81.	Dana	(*impatiently*) So, no, this, I don't have problem with this. Just minus or plus?
82.	Teacher	Well, what is it here? (*points to the +3 on the number line*) (It's) a plus
83.	Dana	Oh, >plus, plus<(goes back to her chair)

Obviously, from Dana's last impatient remarks ([81],[83]) it was **I,** this time, who was not following the interactional routine as she had expected it. My meticulous demonstrations of how to perform the calculations using right and left arrows were deemed by her as superfluous. She was mainly interested in the syntactic attachment of the plus or minus signs to a given number. In this way, Dana was constructing an interactional routine between us of the type:

Initiation: student asks how **to do** the mathematics (how to manipulates the signs)
Response: teacher answers with a (preferably short) answer

Pretty soon, it seemed both of us were adhering to this interactional routine. The next episode took place only a few minutes later:

Turn	Speaker	What is said (what is done)
129. 130.	Dana	Tell me if I was right, OK? (*walks over to the teacher*, *stands next to her*). **I did like this**. This is the zero, right?
131.	Teacher	Which exercise is it?
132.	Dana	Uh.. It's forty eight, all right? (*points to (+3)+(−5))*
133.	Teacher	Yeah. And **what did you get**? Minus two?
134.	Dana	Yeah. I—(*points to the number line*) so I had here [zero]
135.	Teacher	^You're^ right, ^you're^ right
136.	Dana	^Oh!^ A-haa! (*skips to her place*) that's great!

This time, it was Dana who was willing to explicate her mathematizing, yet I nipped this rather rare opportunity in its bud by declaring it as "right". Again, I was acting in disaccord with my declared wish to hear the students think and to develop their mathematical thinking. The fact that Dana was very content with my response ([136]) clarifies how this interactional routine of stating a mathematical narrative and being evaluated as right/wrong was becoming firmly co-constructed between us, even though it was completely contradictory to my initial intentions.

The ease and swiftness with which Dana and I had agreed on this wrong/right interactional routine can be explained in light of the former interactional routines that had *not* been successfully co-constructed between us. Contrary to those (such as in the first interview and in the ladders problem episode), here both the mathematical content was pretty much agreed upon (since I was doing most of the mathematizing) and the identifying messages were quite complementary. Whereas Dana was identified by me as "not knowing" and "in need of instruction", Dana identified herself as "willing to learn" and "in need of directions". Table 2 summarizes the evidence for these direct and indirect identifications as found in the above "adding negative numbers" episode.

The harmonious character of our interactions in the "adding negative numbers" episodes can be seen through the agreement between Dana's identifying narratives and my own 3rd P narratives of her. And yet, with such ritual learning–teaching interactions and dull mathematical communication, Dana had very little chance to expand and develop her mathematical discourse. How could such an unfortunate situation occur, and why was I unaware of it until well after the Course had ended? After all, I was well educated about reform mathematical teaching practices (Boaler, 1997; Lampert, 1990) and genuinely wished to engage my students in meaningful mathematical discussions.

One, easy, answer to this question may be found in the constraints derived by the curriculum practiced in Dana's school, which included mostly algebraic techniques,

Table 2 Indirect identifying found in the "adding negative numbers" episode

Interaction	Implicit identifying $message_{Dana}$Dana	Implicit identifying $message_{Teacher}$Dana
79. Dana: **what do I do**, the closest to zero, for instance plus three, minus three?	I am in need of constant directions	Dana needs concrete, specific directions of "how to do" the calculations
80. Teacher: **You start**, look. **You do**.. uh with the arrows.		
129. Dana: Tell me if I was right, OK?	I am in need of constant external evaluation	Congruent with Dana's story (by matter of silent acceptance).
133. Teacher: .. and what did you get? Minus two?		
135. Teacher: ^You're right^, ^you're right^.	Expression of surprise signals: I'm usually expected to fail.	• 2nd P (implicit): The important thing is that you are right. I am not interested in hearing your mathematizing.
136. ^Oh!^ A-haa! (*skips to her place merrily*) that's great!	Happiness signals: I'm approaching my designated successful identity by being "right".	

demanding skills much more advanced than those Dana had mastered. These constraints left much less freedom for progressive teaching practices than I would have wished to have. And yet, this answer cannot provide the whole picture. As could be seen in the excerpts above, I repeatedly missed opportunities to hear Dana out, or to encourage her autonomous mathematizing, and this had nothing to do with the curriculum. The analysis of all the above excerpts of interaction raises the conjecture that the deviations from normative interactional routines, occurring whenever Dana had opportunity to expose her mathematical reasoning or to engage in explorative mathematizing, led to negative identifying of Dana. This negative identifying led both Dana and me to adhere to those interactional routines that were mutually accepted by both of us—the routines of "give answer, receive evaluation" and "ask for directions, receive concrete explanations". As helpful as these routines were for keeping the smooth flow of interaction between Dana and me, they did not give Dana much opportunity to achieve explorative participation in the mathematical discourse.

3.3 Consequences of the interaction: a convergence of Dana's 1st P and 3rd P identity

In accordance with the mutual adaptation to shared interactional routines, Dana's stories of herself as a mathematical student made a similar convergence towards those I was telling of her. Unfortunately, this meant that by the end of the Course, Dana was telling a story of herself as much less competent in mathematics than she had told at the beginning. To capture this difference, Table 3 compares her highest order identifying statements (general participation evaluations and stable attributes) at the first and final interview.

Interestingly, Dana's stories did not only change regarding her current (7th grade) performance. They also changed regarding how things *used to be* in her past. Thus, from claiming at the first interview that she didn't have much problem with mathematics in elementary school (Table 3 line 2), she now claimed she had always known she was not good in mathematics. This phenomenon, where narratives of past history change over time is well known, especially in young adolescents (Habermas & Bluck, 2000; McAdams, 2001). However, it is notable that this change evolved so that it converged with my own 3rd P story of Dana.

Table 3 A comparison of Dana's identifying in the first and final interview

	First interview	Final interview
1.	(Q: How are you in math now?) Uhhh^ I don't know really (smiles shyly, looks down) [88]	(Q: So, how do you summarize the current year in math?) **Bad**. … 'cause **all the tests and quizzes.. were bad**.
2.	(Q: *Did you have problems with math* in previous years?) *No*. In 6th grade, **decimal numbers, that's really easy**.. to move the dot from one place to another [110]	I've always known **I'm not.. good at it**(at math)[109] In 1st–4th grade there was all this stuff of add and subract.. **and that was ^easy^… then (from 5th grade on) it became tough**. And I don't think I made an effort 'cause **it was really tough**. [359–361]
3.	With algebra, for instance, (*chuckles*) I don't get along very well [84].	*My problem* is that I- I don't know how to do-, I can do the exercise, but, I dunno. After a while I can't remember how to do it… Lots of times I can do it, just if (someone) helps me, somehow. If (they) show me how to do it. [35–38]

Underlined general participation evaluations, *italicized* stable property attributes, *in bold* implicit identifying

4 Summary and conclusions—the interactive mechanism perpetuating failure in Dana's case

At the time of the Course, it seemed that everyone in Dana's case was doing the best they could to help Dana advance in her mathematical learning. Dana herself was well motivated, she was receiving special instruction tailored to her skills, additional instruction from her school teacher, and as much attention as she wished to get from me, her Course teacher. And yet, nothing was improving in her mathematical skills. On the contrary, the gap between her current and expected performance was continually growing. As can be seen in Appendix, Table 5, Dana failed completely on almost all the tasks she was requested to perform in the final mathematical interview at the end of the course. All these tasks were either replicated from the (most simple) tasks of the initial interview, or simplified tasks taken from the 7th-grade curriculum, that were taught and practiced during the Course. Such failure, after intensive instruction, could perhaps serve as a classical example of a learning disabled child, who, despite intervention, does not improve her achievements.

And yet, the close examination of Dana's case revealed this conclusion could hardly be drawn, considering the lack of real opportunities for Dana to participate in the mathematical discourse in an autonomous, explorative, way. Without such opportunities, Dana was restricted to ritual rule following, a preliminary stage of participation (Sfard & Lavie, 2005) that should always be succeeded by explorative negotiation of mathematical meaning if the new mathematical skills are to be individualized. Obviously, Dana had not participated exploratively in mathematical discussions from a very early stage of her schooling. However, what might escape the eye of those who look at Dana solely through the mathematical lens are the *alternative* forms of participation Dana had developed during the years in which she was restricted to ritual rule following.

In that sense, a student never stays still. Something is always learned, whether it is what the teacher intends it to be, or not. In Dana's case, the main thing that was learned from mathematical interactions was how to identify herself *as if* she was a participant in the discourse. Relying on what she was seeing in the activity of fully participating students, Dana attempted to signal she was, too, such a participant: she nodded at the right times, asked inquisitive questions, and attempted to explain and convince her fellow students with her mathematical narratives. In other words, she attempted to follow the interactional routines as best as she could, at least in their identifying content. And yet, because of her mathematical tool kit being appropriate at most for 2nd-grade discourse, she could not adhere fully to these routines because the mathematical content of her words was mostly inappropriate.

The result of this oddness was that Dana was often excluded from any significant mathematical interaction. It is important to stress that it was not Dana's *mathematizing* that caused that, at least in my interactions with her. As most teachers, I was prepared for a different, less developed mathematical discourse coming from my students. After all, this was what teaching was all about. What repeatedly caused this exclusion were the well concealed and non-deliberate *identifying* processes that were occurring between us.

Why was Dana consistently violating the interactional routines to which I was accustomed by evaluating herself differently from how I did? This question may have several answers. First, she hardly had any tools to evaluate her mathematizing correctly. Having developed for herself such an idiosyncratic mathematical discourse, the routines of which were so far removed from the normative ones, she had little chance of evaluating her actions in the same manner that I, or other experts, would. Second, Dana's designated identity as a full participant in the lessons was probably very strong. The disparity between this 1st P designated identity, and the current 3rd P identity (as authored by me) was simply too wide to be accepted by Dana. In fact, the study probably had caught Dana in a *transitional* stage, where she was still not willing to endorse her 3rd P identity as told by all the narrators around her (or perhaps she was not fully aware of it yet).

Students are often somewhat overconfident about their mathematical performance (Pajares & Kranzler, 1995), yet rarely is their 1st P mathematical identity so divergent from their 3rd P identity as in Dana's case at the beginning of the Course. Despite its oddness, Dana's case is important for several reasons. First, by standing out in its peculiarity, it unravels the mechanisms of student–teacher identifying interactions that are usually so routinized that they escape our eyes. Second, it demonstrates how a student, even a very enthusiastic and motivated one as Dana, will come to endorse, at the end, the 3rd P narratives told around her about her mathematical ability. Finally, this case exemplifies how these processes of mutual adherence to certain interactional routines are beyond the control of one of the participants alone, especially if gone unnoticed. It thus strengthens findings from previous studies showing that interactional routines in the classroom are co-constructed by the students as well as the by the teacher, and that therefore changing them is beyond the sole control of the teacher (Lefstein, 2008).

To this day, I do not know if I could have succeeded in changing Dana's ritual participation in mathematical discourse, even if I had been aware of the identifying processes that were taking place between us. Only future research, where such processes would be revealed during the study itself and change would be actively introduced, will show if and how interactional routines between a teacher and a student can be changed.

4.1 Theoretical implications

Some of the phenomena described in this case have been treated by one strand of research or another in the past. For instance, numerous studies have dealt with self-fulfilling prophecies that influence students to become what their teacher expects of them (Brophy, 1983; Jussim, 1989). Social psychologists have attempted to explain the psychological mechanisms standing behind the self-fulfilling prophecy in terms of the inner processes (expectancies and perceptions) that the perceiver and the actor undergo during an interaction (Darley & Fazio, 1980). The present framework, based on the communicational framework, the ideas of

Wittgenstein (1953) and discursive psychology (Harré & Gillette, 1994), frees us from reliance on such untenable inner processes, and enables us to study these phenomena directly from the discourse that takes place in class.

The use of identity as a unifying concept for examining the relationship between the social and the individual (Wenger, 1998) has proven useful for an increasing number of studies in the domain of mathematics education (Andersson, 2011; Black, 2004a; Cobb et al., 2009; Ingram, 2011; Lange, 2009; Nasir, 2002; Solomon, 2007; Wood, 2008). The uniqueness of the method of analysis presented in this paper is in two aspects. First, it suggests a set of operational tools for extracting identifying activity from real life classroom activity, thus freeing the researcher from relying solely on interviews for gaining insight into students' identities. Second, it enables analyzing identity and mathematical activity using a unified set of conceptual tools. In this way, one can move from speculating about what the connections between identity and mathematical skills are to actually examining the mutual influence between identifying and mathematizing.

In the present study, this method yielded surprising findings about the connection between identity and mathematical performance. If at the outset of examining the data I was sure that the connection would be found mainly between the 1st P narratives of Dana and her mathematical performance, the analysis revealed that the 3rd P narratives that I was telling about her had an impact which was no less significant in the perpetuation of her mathematical difficulties. The implications of these findings lie in highlighting the necessity of taking into account the social and affective, as well as the cognitive, aspects of learning difficulties in mathematics. Only with such a holistic account can we see that such difficulties, which have been studied so extensively by cognitive and developmental psychologists (Geary, 2004; Butterworth, 2005; Ginsburg, 1997), may be socially, and not only individually, constructed.

The communicational view can also help us notice the ways in which our language impacts on the treatment of children who are labeled as learning disabled. This treatment is part of the reifying language in which we talk about mathematical disabilities. Take for example the definition of dyscalculia as given in a glossary of a paper talking about "Evidence in Education" which reviews research results that are relevant for education and instruction (Steedly, Dragoo, Arefeh, & Luke, 2008): "Dyscalculia—a form of learning disability that **causes** an individual to have difficulty in understanding concepts of quantity, time, place, value, and sequencing, and in successfully manipulating numbers or their representations in mathematical operations" (My emphasis, p. 10). The nominalization of "learning disability" and the use of the word "causes" conceals the possibility that dyscalculia may be a *product* of a certain human made process whereby a child with persistent difficulties in mathematics (and other cognitive measures) was given a certain diagnostic label. Once the objectification of this process has been achieved, it is easy to talk about the dyscalculia as some physical entity that causes certain behaviors (and is not the reification of them), similarly to a fractured bone that causes pain and movement constraints.

The implications of this objectification of disability are far from trivial. Take for instance the following conclusion, made by Jones et al. (1997) who compared the success of reform programs of instruction with direct instruction in populations of students with learning disabilities (LD).

> The constructivist perspective, though intuitively appealing, is currently unsupported by empirical research and is logically inadequate for the task of teaching adolescents **with LD**. … The premise that secondary students **with LD** will construct their own knowledge about important mathematical concepts, skills, and relationships, or that in

> the absence of specific instruction or prompting they will learn how or when to apply what they have learned, is indefensible, illogical, and unsupported by empirical investigations. (p. 160–161, my emphasis)

Such recommendations for teaching students with LD are exemplified in one of a series of recommendations for teachers of learning disabled students in algebra: "Provide guided practice before independent practice so that students can first understand what to do for each step and then understand why" (Witzel, Smith, & Brownell, 2001, p. 104).

No mention is made in these recommendations for students with LD to engage in explorative learning of mathematics. I do not wish to argue with the empirical findings of the well-respected scholars cited above. The problem lies in the permanence of the disability title and the apparent disregard for the social and affective processes that may be (at least partially) responsible for its development in the first place. It also lies in the fact that these direct instruction methods, as efficient as they might be in producing students who are well trained in carrying out routines, may perpetuate ritual participation in mathematics and failure in the long run.

Of course, the method of analysis presented in this paper in general and the present case study in particular are not without their limitations. One obvious limitation lies in it being only one case of a single student, and a very exceptional student indeed. Future research on larger groups of students with learning difficulties would be needed to generalize any conclusions about how such students cope with the ever-growing gap between their own mathematical discourse and the discourse in which they are designated to participate. And yet, what *can* be learned from this study is the *existence* of interactional routines such as those displayed in this case, and their effect on the construction of certain mathematical identities. Most important, the lesson to be learned from this case is that disability in mathematics may be re-constructed in every interaction between the student and the teacher, a matter that would be very important for teachers of children who experience difficulties in mathematics to remember.

4.2 Meta-reflection—how the present method of analysis enabled changing my 3rd P story of Dana

This paper would not be complete without a word of reflection about the process that I, as a teacher, underwent through this study. No doubt, the process was not pleasant. Discovering that I, as a teacher, have a responsibility for my student's failure is not an easy experience. Yet this discovery is precisely what offers teachers the opportunity to change and improve our teaching. What was it, in the present method, that enabled this change? More precisely, what was it that enabled the profound change in my 3rd P story of Dana, in such a way that I could see my own responsibility for its construction?

Let me first say, that this change was not achieved easily. The process of identifying students in a certain way, according to a certain social category is such an inherent, indeed necessary, part of our discursive practices, that changing it is very difficult. The present method, which focuses on separating between the *mathematical* content of the teacher-student interaction and the *subjectifying* content of it, helped me to distill the identifying stories that I was authoring about Dana. Those became evident when I started looking at Dana's mathematizing from a more neutral stance, as a participant in a *different* kind of mathematical discourse, not a deficient kind.

From this point of view, Dana's claims were suddenly not as ridiculous as I had thought them to be. They had some logic to them, even though this logic was very different from mine. Once I realized that, I could see that my disregard of Dana's attempts to participate in class was not a result only of *what* she was saying but of *who* was saying it, in other words, of how I was identifying Dana.

And yet, this realization in itself was not enough. After all, something *was* odd in how Dana acted in class, though I found it surprisingly difficult to explicate, now that I was viewing her discourse from a more neutral viewpoint. The necessity to explicate this oddness, and to connect it to how my former identification of Dana as "clueless" was initially constructed, brought me to unearth the interactional routines that we were both following during the Course in such an unreflective manner.

Acknowledgments I am grateful to Prof. Anna Sfard, my advisor for the present study. In addition, I wish to express my thanks to the Mandel Scholars program for PhD students and the Golda-Meir foundation of the Hebrew University for having provided financial support for the writing process. Finally, I wish to thank Dr. Adam Lefstein and the anonymous reviewers for their useful comments on the first drafts of this manuscript.

Appendix

Table 4 Dana's first assessment—10/2/08

Task	Performance
1. Translate words to numbers: "thirty nine thousands and eight"	Correct
2. "write sixteen million, six hundred and five thousand, and nine"	Incorrect
3. Pronounce 7,086	Correct
4. Pronounce 80,008,800	Incorrect
5. Pronounce 20,304	Correct, after assistance
6. Calculate 5,000–2	Incorrect
7. Calculate 96+7,935	Incorrect
8. Calculate 778–245	Correct, after assistance
9. Estimate 1,531+2,147	Correct, after assistance
10. What number would I get if I add 4 to the tens numeral in the number 1,639	Partially correct (writes 1,679, says "sixteen seventy nine"), with assistance
11. What number would I get if I reduce 10 from the number 100,000?	Incorrect
12. Calculate 3×5	Correct
13. Calculate 5×30	No answer, gives up in the middle.
14. Calculate 8×4	Correct, after assistance
15. Calculate 80×4	Incorrect
16. Calculate 4×80	Incorrect
17. Calculate 25×99	Incorrect
18. Shirts and pants problem	Incorrect
19. Express the shaded part of the drawing as a fraction (7 circles, 2 of them shaded black)	Incorrect
20. Write(>, <, or =) 1/5 [] 1/7	Incorrect

Table 4 (continued)

Task	Performance
21. Write(>, <, or =) 3/7 [] 5/7	Correct (not justified)
22. Write(>, <, or =) $2\frac{3}{4}$ [] $2\frac{2}{9}$	Incorrect
23. Write(>, <, or =) $\frac{6}{12}$ [] $\frac{1}{2}$	Incorrect
24. Solve $\frac{1}{2}+\frac{1}{10}$	Incorrect
25. Solve 12 $\frac{1}{2}+10\frac{1}{4}$	Incorrect
26. Solve $\frac{2}{3}\times 9$	Incorrect
27. Solve $\frac{7}{40}\div\frac{1}{4}$	Incorrect
28. New comers problem	Correct, with assistance
29. Translate to a simple fraction: 0.7 =	Incorrect
30. Translate to a simple fraction: 1.25 =	Incorrect
31. Translate to a simple fraction: 0.05 =	Incorrect
32. Translate to decimal: 8/10 =	Incorrect
33. Translate to decimal 2/5 =	Incorrect
34. Translate to decimal 1 3/4 =	Incorrect
35. Solve 199+1.4 =	Incorrect
36. Give change for 63.75 NIS from 100 NIS.	Incorrect
37. School area problem	Incorrect
38. You have deposited in the bank 28 bills of 100 Shekels, 17 coins of 10 Shekels each, 47 coins of 1 Shekels. How many Shekels have you deposited?	Incorrect
Total correct, without assistance	4/38=10.5 %
Total correct, (including with assistance)	9/38=23.7 %

Table 5 Dana's final assessment—24/6/08

Task	Performance
1. "write sixteen million, six hundred and five thousand, and nine"	Incorrect
2. Calculate 8×4	Correct, with assistance
3. Calculate 80×4	Incorrect
4. Calculate 4×80	Correct, with assistance
5. Write(>, <, or =) −3/7 [] −5/7	Incorrect
6. Write(>, <, or =) −1 [] 2	Correct
7. (−5)–(+2)=	Incorrect
8. (+10)–(−5)=	Not attempted
9. (−20)*(+3)=	Not attempted
10. Find the substitution set of $\frac{20}{5-x}$	Incorrect
11. Substitute x=−2 in the expression 3×+6	Not attempted
12. Combine like terms 5X+7Y+3-Y+9X-1	Partially correct
13. X+4=7	Correct (incorrect procedure)
14. 4×+5=2×+7	Incorrect
15. 8(3×–4)+(3–4×)4=36	Incorrect
Total correct (without assistance, correct procedure)	1/15=6.7 %
Total correct (with assistance)	4/15=26.7 %

References

Anderson, K. T. (2009). Applying positioning theory to the analysis of classroom interactions: Mediating micro-identities, macro-kinds, and ideologies of knowing. *Linguistics and Education, 20*(4), 291–310.

Andersson, A. (2011). *Engagement in education: Identity narratives and agency in the contexts of mathematics education*. (Unpublished doctoral thesis). Aalborg University: Aalborg.

Ben-Yehuda, M. (2003). *Using discourse analysis to investigate arithmetical thinking processes of students with learning difficulties*. (Unpublished doctoral thesis) (Hebrew). University of Haifa: Haifa.

Ben-Yehuda, M., Lavy, I., Linchevski, L., & Sfard, A. (2005). Doing wrong with words, what bars students' access to arithmetical discourses. *Journal for Research in Mathematics Education, 36*(3), 176–247.

Bishop, J. P. (2012). She's always been the smart one. I've always been the dumb one: Identities in the mathematics classroom. *Journal for Research in Mathematics Education, 43*(1), 34–74.

Black, L. (2004a). Differential participation in whole-class discussions and the construction of marginalised identities. *Journal of Educational Enquiry, 5*(1), 34–54.

Black, L. (2004b). Teacher–pupil talk in whole-class discussions and processes of social positioning within the primary school classroom. *Language and Education, 18*(5), 347–360.

Boaler, J. (1997). Reclaiming school mathematics: The girls fight back. *Gender and Education, 9*(3), 286–306.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics education* (pp. 171–200). Westport: Ablex.

Brophy, J. E. (1983). Research on the self-fulfilling prophecy and teacher expectations. *Journal of Educational Psychology, 75*(5), 631–661.

Brubaker, R., & Cooper, F. (2000). Beyond "identity". *Theory and Society, 29*(1), 1–47.

Butterworth, B. (2005). Developmental dyscalculia. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 455–468). New York: Psychology Press.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning* (2nd ed.). Portsmouth: Heinemann.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education, 40*(1), 40–68.

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics, 23*, 99–122.

Darley, J. M., & Fazio, R. H. (1980). Expectancy confirmation processes arising in the social interaction sequence. *American Psychologist, 35*(10), 100–122.

Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour, 20*(1), 43–63.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131–147.

Eisenmann, T., & Even, R. (2011). Enacted types of algebraic activity in different classes taught by the same teacher. *International Journal of Science and Mathematics Education, 9*, 867–891.

Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics, 63*(2), 209–226.

Fletcher, J. M., & Vaughn, S. (2009). Response to intervention: Preventing and remediating academic difficulties. *Child Development Perspectives, 3*(1), 30–37.

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology, 97*(3), 493–515.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4–15.

Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 253–267). New York and Hove: Psychology Press.

Gee, J. P. (2000–2001). Identity as an analytical lens for research in education. *Review of Research in Education, 25*(99–125).

Ginsburg, H. P. (1997). Mathematics learning disabilities: A view from developmental psychology. *Journal of Learning Disabilities, 30*(1), 20–33.

Goffman, E. (1967/2005). *Interaction ritual: Essays in face-to-face behavior/Erving Goffman; with a new introduction by Joel Best*. Transaction Publishers: New Brunswick.

González, J. E. J., & Espínel, G. A. I. (1999). Is IQ-achievement discrepancy relevant in the definition of arithmetic learning disabilities? *Learning Disability Quarterly, 22*(4), 291–301.

Graven, M. (2011). Creating new mathematical stories: Exploring potential opportunities within maths clubs. In H. Venkat & A. Assien (Eds.), *Proceedings of the Seventeenth National Congress of the Association for Mathematics Education of South Africa* (pp. 161–170). Johannesburg: AMESA.

Habermas, T., & Bluck, S. (2000). Getting a life: The emergence of the life story in adolescence. *Psychological Bulletin, 126*(5), 748–769.
Harré, R., & Gillette, G. (1994). *The discursive mind*. Thousand Oaks: Sage.
Harré, R., & van Langenhove, L. (Eds.). (1999). *Positioning theory*. Oxford: Blackwell.
Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education, 21*(1), 33–46.
Heyd-Metzuyanim (2012). *Emotional aspects of learning mathematics – How the interaction between identifying and mathematizing influences the effectiveness of learning*. (Unpublished doctoral thesis). University of Haifa: Haifa.
Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom—learning mathematics as interplay between mathematizing and identifying. *International Journal of Educational Research, 51–52*, 128–145.
Ingram, N. (2011). *Affect and identity: The mathematical journeys of adolescents*. (Unpublished doctoral thesis). University of Otago: Dunedin.
Jones, E. D., Wilson, R., & Bhojwani, S. (1997). Mathematics instruction for secondary students with learning disabilities. *Journal of Learning Disabilities, 30*(2), 151–163.
Jussim, L. (1989). Teacher expectations: Self-fulfilling prophecies, perceptual biases, and accuracy. *Journal of Personality and Social Psychology, 57*(3), 469–480.
Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7*(3), 164–177.
Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29–63.
Lange, T. (2009). *Difficulties, meaning and marginalisation in mathematics learning as seen through children's eyes*. (Unpublished doctoral thesis). Aalborg University: Aalborg.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.
Leander, K. M. (2002). Locating Latanya: The situated production of identity artifacts in classroom interaction. *Research in the Teaching of English, 37*(2), 198–250.
Leder, G. C., & Forgasz, H. J. (2006). Affect and mathematics education. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 403–427). Rotterdam: Sense Publishers.
Lefstein, A. (2008). Changing teacher practice through the National Literacy Strategy: A micro-Interactional perspective. *American Journal of Education Research, 45*(3), 701–737.
Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education, 29*(5), 533–554.
Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education, 30*(5), 520–540.
Mazzocco, M. M. (2005). Challenges in identifying target skills for math disability screening and intervention. *Journal of Learning Disabilities, 38*(4), 318–323.
Mazzocco, M. M., & Myers, G. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of Dyslexia, 53*, 218–252.
McAdams, D. P. (2001). The psychology of life stories. *Review of General Psychology, 5*(2), 100–122.
McCall, G. J., & Simmons, J. L. (1966). *Identities and interactions*. New York: The Free Press.
McDermott, R. P. (1993). The acquisition of a child by a learning disability. In S. Chaiklin & J. Lave (Eds.), *Understanding practice* (pp. 269–305). Cambridge: Cambridge University Press.
McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: Macmillan Publishing Company.
Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge: Harvard University Press.
Montague, M. (2007). Self-regulation and mathematics instruction. *Learning Disabilities Research and Practice, 22*(1), 75–83.
Nasir, N. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical Thinking and Learning, 4*(2), 213–247.
Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology, 24*, 124–139.
Pajares, F., & Kranzler, J. (1995). Self-efficacy beliefs and general mental ability in mathematical problem-solving. *Contemporary Educational Psychology, 20*(4), 426–443.
Roth, W. M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning*. Rotterdam: Sense Publishers.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education, 20*(4), 338–355.
Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the Learning Sciences, 16*(4), 565–613.
Sfard, A. (2008). *Thinking as communicating*. New York: Cambridge University Press.
Sfard, A., & Lavie, I. (2005). Why cannot children see as the same what grown-ups cannot see as different?—early numerical thinking revisited. *Cognition and Instruction, 23*(3), 237–309.
Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22.
Shalev, R. S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y., et al. (2001). Developmental dyscalculia is a familial learning disability. (Statistical data included). *Journal of Learning Disabilities, 34*(1), 59–65.
Solomon, Y. (2007). Experiencing mathematics classes: Ability grouping, gender and the selective development of participative identities. *International Journal of Educational Research, 46*(1–2), 8–19.
Steedly, K., Dragoo, K., Arefeh, S., Luke, S. D. (2008). Effective mathematics instruction, evidence for education. *3*(1). Downloaded on October, 12, 2012 from http://nichcy.org/wp-content/uploads/docs/eemath.pdf.
Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics, 26*, 275–298.
Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press.
Wittgenstein, L. (1953). *Philosophical investigations* (trans: Anscombe, G.E.M.). Oxford: Blackwell.
Witzel, B. S., Smith, S. W., & Brownell, M. T. (2001). How can I help students with learning disabilities in algebra? *Intervention in School and Clinic, 37*(2), 101–105.
Wood, M. B. (2008). *Mathematizing, identifying, and autonomous learning: Fourth grade students engage mathematics*. (Unpublished doctoral dissertation). Michigan State University: East Lansing, MI.
Wood, M. B., & Kalinec, C. A. (2012). "What? A Black brother can't talk?" Possibilities for learning mathematics in small group interactions. *International Journal of Educational Research, 51–52*, 109–127.
Wortham, S. (2006). *Learning identity: The joint emergence of social identification and academic learning*. New York: Cambridge University Press.