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Shifting Mathematical Authority from Teacher to Community

"Mr. Webel, is this right?"

As a high school teacher, I was asked this question daily. I never knew quite how to answer. On the one hand, I wanted my students to make sense of and provide mathematical reasons for their answers rather than simply take my word for it. On the other hand, what was I supposed to say—"I refuse to answer" or "I don't know"? These responses felt disingenuous, as if I were keeping knowledge from students simply for the purpose of watching them struggle.

This dilemma is not uncommon. Research shows that students generally view mathematics teachers as the final authority on mathematical correctness, the sole arbiter of whether or not answers are right (Muis 2004). Such reliance is problematic. Students who always defer to their teachers are failing to make sense of their own solutions. Moreover, they thus position themselves as outsiders with respect to mathematical thinking and discourse (Amit and Fried 2005); they define mathematics as something that only very smart people can do and "view themselves as copiers of others' mathematical knowledge" (Muis 2004, p. 328).

The implications of this view go beyond the classroom setting. If mathematics cannot be understood, only copied, then students have locked themselves out of any mathematical situation in which they do not already know exactly what to do or in which an expert is not immediately available.

SHIFTING THE AUDIENCE

In my classroom, I could picture the kind of discourse I wanted—students sharing ideas, arguing over and extending one another's thinking, trying out ideas before they knew whether or not they would work, and deciding on their own whether their answers made sense—but I did not know how to make this discourse happen. Part of the problem was that I needed my students to be engaged, but I was not sure whether I could engage them.

The primary problem with my teaching, I now believe, was that it did not make use of the community that existed in my classroom. Students learn much more in mathematics class than just mathematics; they learn to coexist, to fit in, to *belong*. My instruction did not tap into students' need to find their place in the classroom community. For example, when I asked my students to justify their mathematical solutions, I was establishing myself as the only audience for those justifications. Mathematical justification is not only about understanding why a solution is correct; it is also about *convincing others* (Harel and Sowder 2007). From my students' perspective, convincing me was pointless; I already knew whether or not their solutions were correct.

If I really wanted to change students' dependence on my authority, I needed to shift the audience and get students talking to someone whom they could convince—one another. When students are required to convince their classmates, sharing ideas becomes an important way of fitting in because doing so can help the group decide what is right. Proving becomes more than merely another exercise to be

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completed and verified by the teacher; it becomes a way of establishing oneself as a member of the community. Under these conditions, knowledge of what is mathematically legitimate is not determined by relying on the teacher's authority; it is established though a community-driven process of argumentation, negotiation, revision, and consensus.

MAKING SHARED AUTHORITY A REALITY

What can teachers do to shift the audience for mathematical talk and establish mathematical legitimacy as a communal concern? As a graduate student, I had the opportunity to spend several days visiting a high school, comparing the different ways in which the same lesson was enacted in the classrooms of four teachers. I observed five ninety-minute lessons enacted by each teacher. My goal was to try to understand how different teaching strategies led to different kinds of student interaction.

Walshaw and Anthony (2008) make it clear that student outcomes are not caused by teachers' practices so much as *occasioned* by them. In other words, whereas teachers cannot manufacture discourse on their own, they can influence the circumstances in which mathematical discourse takes place. For example, teachers can affect the timing, length, topic, and participation structure of class discussions as well as their own role in facilitation. Here I describe differences in the structure of collaborative learning formats in two classrooms, discuss how these structures did or did not seem to help shift the locus of authority from teacher to community, and reflect on how this experience has affected my own teaching practices.

TWO EXAMPLES: MR. NEAL AND MS. CRAWFORD

Mr. Neal and Ms. Crawford (these names are pseudonyms) worked in the same school, teaching the same course from the same reform-oriented textbook, *Core Plus Mathematics* (Hirsch et al. 2008). Both teachers wanted to make sure that each student understood mathematical ideas on his or her own, independent of an expert authority. They both used group work regularly in their ninety-minute class periods and expressed an interest in having students learn from one another. Although these two teachers' views on teaching seemed similar, the nature of the discourse in their classrooms was noticeably different.

In Mr. Neal's classroom, students often engaged in lively mathematical discussion; they posited, challenged, and revised solution strategies without relying on Mr. Neal for approval. During one lesson, students spent several minutes arguing about whether or not an exponential growth factor could be considered a slope. The students were visibly engaged—they were attentive, talking to one another, offering their arguments, and explaining their reasoning while Mr. Neal and his mathematical authority faded into the background. During these class discussions, students' comments drove the discourse, and the need to understand and come to agreement was evident in the students' level of participation.

Ms. Crawford too put much effort into getting her students to share their ideas and engage in deep mathematical thinking. For example, during group work, Ms. Crawford stressed specific roles for individual students, attempting to ensure that the group worked together. "Quality controller," she announced one day, "Will you please make sure that everyone [in your group] is holding on together, because this is not a frivolous assignment." During whole-class discussions, Ms. Crawford often asked students to vote on whether or not they agreed with what another student had said. She sought to encourage participation by waiting until several hands were raised before calling on students, at times calling on those whose hands were not raised. In an effort to provide a safe environment for sharing ideas, she allowed the option of "phoning a friend" for students who were unable to provide a response. However, despite all these attempts to help students share their mathematical thinking, Ms. Crawford was unable to produce the kind of student-to-student interaction evident in Mr. Neal's classroom.

Discussions in Ms. Crawford's class did not develop into autonomous, student-led discourse. Students presented their solutions to various problems, but these presentations did not blossom into the kind of legitimate debate about ideas that I witnessed in Mr. Neal's class. After students presented their solutions in Ms. Crawford's class, she would, at times, press them to explain their answers, but these explanations were directed back to her rather than to their peers. Students rarely challenged others' solutions, and most seemed content to wait for Ms. Crawford to say whether or not the answer was correct.

The differences between these two classrooms, with respect to the locus of authority, the audience for ideas, and the quality of student discourse cannot be easily explained. In the following sections, I identify and describe how particular aspects of the two teachers' practices may have influenced the way in which their students participated in mathematical discourse.

GROUP WORK AND WHOLE-CLASS DISCUSSIONS

One classroom practice that seemed to play a significant role in shaping discourse was the way in which group work was structured in each class. In Ms. Crawford's class, students spent long stretches of class time working in groups. For example, during one particular class session, students were working on a compound interest investigation (Hirsch et al. 2008). Ms. Crawford gave her students forty-five minutes to work in groups on six problems, an assignment that took them to the end of the class period. For the same lesson on compound interest, rather than assigning all six problems from the investigation at once, Mr. Neal gave his students three minutes to work on problem 1. Next, he called them together for a whole-class discussion to have students share their ideas for about sixteen minutes. Students then worked in groups on problems 2 and 3 for ten minutes before coming together for another whole-class discussion. Mr. Neal continued this pattern for the rest of the class, allowing sometimes as few as two minutes for students to work in groups before calling them back to whole-class discussions.

This strategy of having students work in groups for short intervals (rarely more than fifteen minutes at a time) was a regular part of Mr. Neal's class structure, whereas Ms. Crawford typically gave students thirty to forty minutes at a time to work in groups. Although this was not the only factor contributing to the difference in student discourse between these two classrooms, this strategy seemed to afford Mr. Neal opportunities for distributing mathematical authority.

WHY DURATION MATTERS

When groups of students are given an extended period of time to complete a task, getting stuck or going down the wrong path can put them considerably behind the rest of the class. Often the first thing students will do when unsure about a solution strategy is to seek help from the teacher. Rather than tinkering with ideas and testing solutions and asking one another questions, they become dependent on the teacher for making sure that they do not make mistakes. They sit, paralyzed and helpless, waiting for the teacher to come and provide guidance.

In classrooms where the teacher breaks the lesson into small segments of group work, students are more willing to try strategies before they know whether these will work and are less likely to feel that they need to obtain the teacher's approval before trying out an idea. For these students, getting stuck or going down the wrong path is not a major setback; it simply means that, in a few minutes, they will have a chance to come together as a class and compare their solution with other groups' solutions. When students in Mr. Neal's class got stuck, he would often respond, "Okay, well, something may be wrong with what you are doing, but we'll figure it out." He was setting up the whole-class discussion among the members of the community as the primary resource for resolving the confusion that arose in individual groups.

In many classrooms, as teachers circulate among student groups during investigations, they find themselves doing a lot of mathematics for students who are stuck, lowering the cognitive demand and reasserting their position as mathematical authority. By giving students short periods of time to work, teachers can more easily avoid taking over mathematical thinking; the imminent class discussion becomes the standard forum for resolving confusion. Indeed, Ms. Crawford was twice as likely as Mr. Neal to use questions that funneled students to a particular answer when giving mathematical help (32% of her comments to groups, compared with16% for Mr. Neal). Rather than using group work as a time to help students make progress, teachers can use it to *listen* to students talk and to understand what they are thinking rather than immediately moving in to "fix" student misconceptions (Davis 1997).

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EFFECT ON WHOLE-CLASS DISCUSSIONS

Breaking up group work into short segments also has the potential to enhance the discourse that takes place during whole-class discussions. In Mr. Neal's class, deciding which solution was correct was a community task. Instead of giving polished, "final draft" presentations—as was usually the case in Ms. Crawford's class—students in Mr. Neal's class shared their in-process thinking. He asked students to explain, rephrase, and interpret classmates' solutions without indicating which solutions were correct. Rather than simply taking notes, students contributed their own ideas to the conversation by disagreeing, adding detail, or sharing an alternative approach.

Mr. Neal believed that whole-class discussions were an opportunity for students to "discuss what we have learned, hopefully extending our knowledge of the concept even further through talking and listening to one another." For Mr. Neal, the whole-class discussion was not a time for him to summarize what students were supposed to have learned in small groups; it was an opportunity for students to compare their ideas with those developed in other groups.

CHANGING TEACHING PRACTICES

As teachers, we know that the complex problems of teaching rarely have simple solutions. Merely changing the structure of a class to incorporate short periods of group work is not, by itself, likely to transform classrooms into learning communities that share mathematical authority. Certainly, teachers can undermine the potential of this strategy by providing too much mathematical scaffolding during group work, by reasserting their authority during whole-class discussions—for example, by summarizing what students were to have learned, immediately correcting student errors rather than letting the rest of the class evaluate, or choosing to present only correct solutions—or by failing to create a safe, respectful atmosphere in which students feel comfortable sharing their ideas. Nevertheless, the structure and integration of collaborative learning formats is an issue with which all teachers committed to Standards-based instruction (NCTM 2000) must wrestle and one that has implications for how mathematical authority is distributed in the classroom (Wilson and Lloyd 2000).

Spending time in these two classrooms has influenced my own teaching in significant ways. When my students work in groups, I am less concerned about providing help and more concerned about planning the next whole-class discussion. I listen for ideas: Which group has uncovered an important misconception that others might need to hear and work through? Which task is eliciting a variety of opinions? Is there a particular point at which several groups are getting stuck? Rather than trying to resolve students' confusion by giving explanations during whole-class discussions, I provide students with opportunities to compare, revise, and combine the ideas that they developed in groups.

When ideas are exhausted or when confusion still exists, rather than stepping in to clarify, I change the format. We go back into groups and work on a new task that extends or reiterates important ideas, or we move to individual tasks, such as journaling about the topic. Changing the format gives students different kinds of opportunities to develop and share their ideas. It helps create and maintain a communal sense of responsibility for understanding ideas and deemphasizes my role as the primary source of mathematical authority.

CONCLUSION

Mr. Neal showed me that sharing authority with students is not a matter of providing less structure; his classes were quite structured. What mattered was how the structure worked to establish the community as the primary audience for students' mathematical thinking. His strategy of breaking up the class into small periods of group work was a key component in creating an atmosphere in which students were less dependent on his authority, were more engaged in comparing, evaluating, and revising one another's solutions, and were more likely to see one another as resources for determining what was mathematically correct. This format provided students with low-risk opportunities to try out their own ideas and gave them social incentives to engage in whole-class discussions.

These observations suggest that making adjustments to the duration and timing of periods of cooperative learning can play an important role in establishing students as capable and confident problem solvers and legitimate contributors to mathematical discourse.

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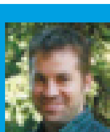

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