Logic of Beauty, Moderation, Rationing and Triage

H.M. Hubey, Professor

Department of Computer Science

Montclair State University, Upper Montclair, NJ 07043

Abstract

A fuzzy logic of triage is given. It is shown that abstract concepts such as beauty, moderation, mediocrity are about triage. It is shown that societal rationing of goods and services also follow the triage law. It is demonstrated that many phenomena in the life and social sciences are best explicable via the de facto standard normalized model making via the use of fuzzy logical concepts. However the corresponding t-norm and t-conorm do not reduce to crisp logic, thus triage is really a more general version of fuzzy logic, one that can serve as a de facto standard for modeling.

Keywords: triage, moderation, fuzzy logic, beauty, taste

1.Introduction to Triage

During wartime the most heavily wounded are not given immediate treatment because they will probably die even if treated. Lightly wounded are not treated immediately because they will not die even if they are not treated. The ones who are treated are those who will probably survive if treated and die if not. This middle-third is where treatment does most good. The basic concept of triage, logic in thirds, occurs often in real life. It is a logic of *middlings*, of *averages* and of *mediocrity.* It is the logic of what in sociology is called the'*herd instinct*', of the need of people to be like others, and to avoid being different despite claiming to be unique. The usage of such negative words to describe triage is purposive because we can just as easily say the same thing in glowing positive terms. For example, we can say that triage is about the common logic of humankind in avoiding *extremes* and *deviancy.* It is about the tendency of humans to consider mediocracy a common and perhaps subconscious goal. In other words the drive to excel and to be in the extremes or the tails of the distributions must be a conscious and purposive activity. These are the middlings. This article is about a logic of middlings, the averages, the normals, not the extrema, not the deviants, not the minima or the maxima. It is about a logic of mediocrity. It is a logic that most people use in real life because in real life most people are mediocre. It is strange how words that all essentially mean the same thing have such variant value-laden emotive content. Indeed, it can be used beyond human societies. There are many cases in real life in which an optimum (minimum or maximum) occurs, although most of these might indeed be related to life in general. For example, there is an optimum temperature, a normal temperature for animal species. There is a temperature of annealing for different grades of steel. There is an optimum even in fuzzy logic in that the change from A to non-A occurs most rapidly in one specific region. The derivative of the truth-valuation function in this case is the ambiguity function. For example perception in forced binary discrimination along a single stimulus can be set up as a differential equation using concepts from fuzzy logic [Hubey,1994];

$$1) \qquad \frac{dP}{dx} = P' = P(1-P)$$

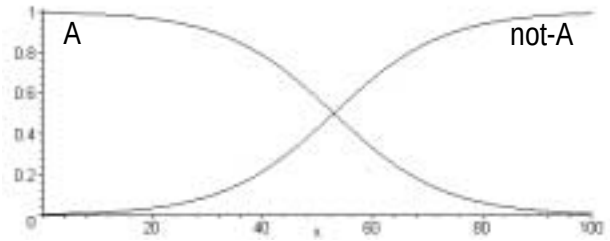

Figure 1: Forced Binary Discrimination

The rhs of the equation can be interpreted as something like distance using its meaning as the conjunction. In other words, the rate of change of perception is zero at the two extremes since clearly at those extremes, the object is either A or $\bar{A}$ (non-A). The most rapid change of perception occurs at the point where $A \wedge \bar{A}$ is maximum, i.e. the point of maximum confusion, Of course this can only be the point at which the derivative P' is maximum. Since the solution of the DE is

2) $$P = \frac{1}{1 + Ke^{-\alpha x}}$$

The derivative is

3) $$P = \frac{K\alpha e^{-\alpha x}}{(1 + Ke^{-\alpha x})^2}$$

and is plotted in Figure 2. In the case of triage, it

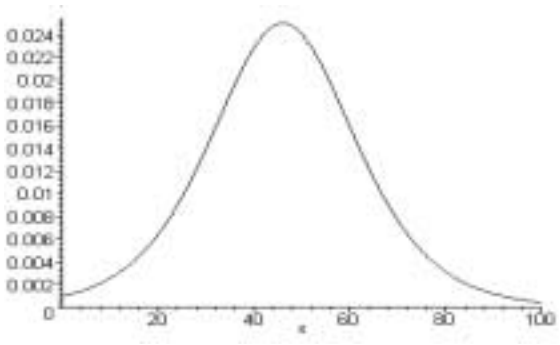

Figure 2: The Ambiguity Function
The meaning of the derivative of the truth valuation

seems as if a ternary logic is required. For example, in the clear case of the war-wounded, the three classes could be {0,1/2,1}. In which case for the implication $W \Rightarrow T$ (*If Wounded, then Treat immediately*), we seem to require a 2-3 logic, i.e. a table of form

T \ W	0	1/2	1
0	1	0	1
1	0	1	0

Figure 3: Truth Table for 2-3 Triage Logic

It is not very clear at this point exactly what all the values should be. It is not even clear that this should be called a fuzzy logic at all since it does not reduce to crisp logic at its boundaries. However, it is a de facto standardized modeling methodology, with a standard normalization and with a reasonably good set of de facto standard operations. One might even say that this state of affairs is even good for fuzzy logic since we can then enrich 'fuzzy logic' by adding more operations. One may even go to the next step and add fuzzy operations to create even better standardized models of reality [Hubey,2000]

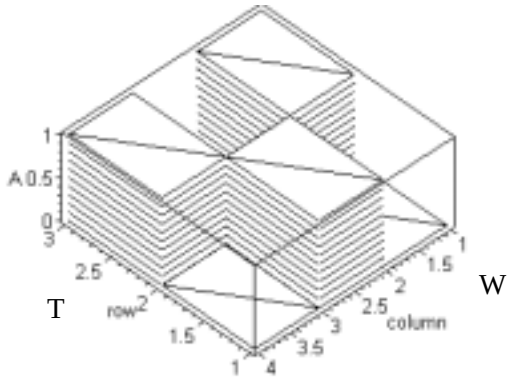

Figure 3: Triage 2-3 Logic

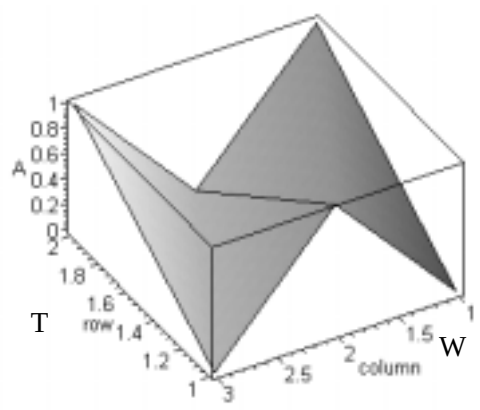

Figure 4: Triage 2-3 Logic; another view

It should be noted that the value of $W \Rightarrow T$ in Figure 4 for T=0 is the negation of the value for T=1 which need not necessarily be true. It is rather unusual to

have a logic in which some variables have 3 values and some have 2, thus normally we would have written Table 1 as 3×3 table as below.

T \ W	0	1/2	1
0	1	$\frac{1}{2}$	0
1/2	0	$\frac{1}{2}$	1
1	0	0	0

Figure 5: Ternary Triage

A plot of this particular rendition of triage is shown in Figure 6.

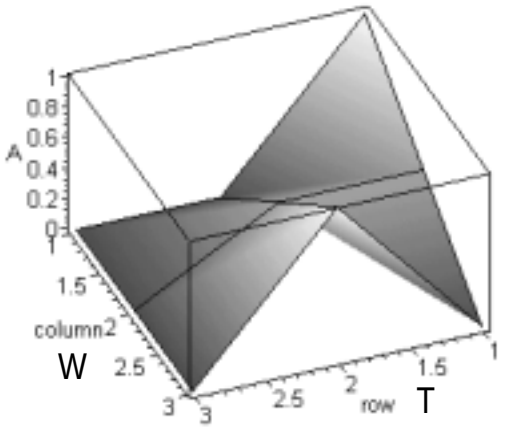

Figure 6: Ternary Triage

It is very strange that triage shows up in so many places that we can barely recognize it. For example, drug dosage is triage logic. Rationing of goods in society is based on triage logic. One can even see this in mundane matters such as licensing. For example, a licence is required to be able to cut hair, but not to bear children. A licence is required to drive a car but not to work in industry with the name *engineer*. A licence is needed to hunt and fish but not to repair cars or computers. In some cases these can be justified by consumer protection needs such as protecting the public from quack physicians, or from those that have not learned how to drive cars. But doesn't the public also need protection from children born to children or to those who can barely take care of themselves? Or even more ominously, we are always told that we have great social ills but no money is being expended on them. This is certainly a great example of the operation of triage. The problems are so complex and so difficult that the public balks at the money spent on programs when there is such small chance of success. Instead the intellectual and mental energy of society is spent on those things which will create some tangible benefit that the masses can see and work for. Rationing, which used to be done directly in command systems (i.e. communism) in socio-economic systems is also done via triage in that there is few of those that are for the richest and the poorest, just as there are fewer rich and fewer poor than the middlings.

2. Derivation of Fuzzy Logic of Triage

The fact that if there are wounded then they should be treated according to the triage precept can be pictured as in Fig. (7) or Fig. (8).

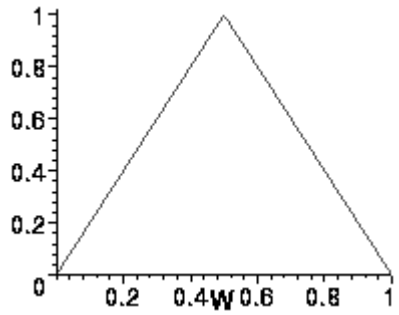

Figure 7: Piecewise Linear Triage valuation

The equations for this are

4) $\qquad T_1 = 2(WU^{-} + (1-W)U^{+})$

where $U^{-} = Heaviside(0.5 - W)$ and $U^{+} = Heaviside(W - 0.5)$ for the piecewise linear (i.e. triangular version) and

5) With $T_1 = -4Z + 1 \qquad Z = (W - 0.5)^2$

for the parabolic version.

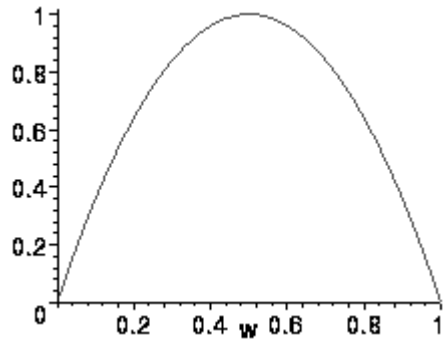

Figure 8: Parabolic Triage valuation

Even a more fitting one is using information theory

$$6)\ T_1 = \frac{-W\log(W)-(1-W)\log(1-W)}{\log(2)}$$

All of these are for the case only if W=1. In other words they encode the rule *"If x is wounded, then treat with degree of membership given by T"* where the case of not-wounded is left unsaid. The maximum immediate treatment is at the middle in all three cases which is what triage is all about. However we need to also know the valuation at W=0 (i.e. no wounded at all). Figure (9) shows this for the piecewise linear case.

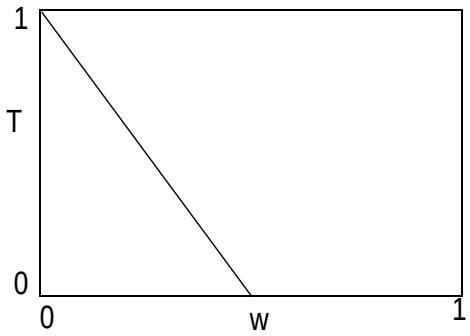

Figure 9: No-Wounded Treatment

We must combine both values in order to produce the implication $W \Rightarrow T$ ("if wounded, then treat"). We need to produce a *smooth transformation* from the values obtained at T_0 to the values obtained at T_1. The equations to achieve this are:

$$7)\quad m(W,T)_L = \begin{cases} 2TW + (1-2W)(1-T) & W < 1/2 \\ 2T(1-W) & otherwise \end{cases}$$

where the $m(W,T)_L$ is the linear material implication. The plot for the fuzzy implication $W \Rightarrow T$ is given in Figure (10).

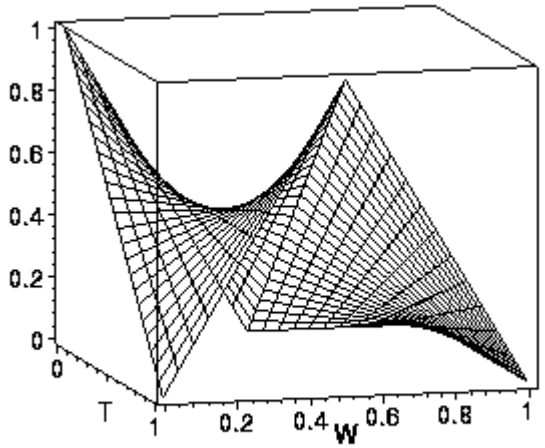

Figure 10: The Implication $W \Rightarrow T$ for Piecewise Linear.

It should be noted that the value is false everywhere except at W=0, and T=0 and at W=1/2 and T=1. Since it does not reduce to crisp logic at the boundaries, this cannot really be called logic in that sense. What it is, however, is a generalized and normalized, and standardized way of modeling phenomena in the real world, especially the branches of study, life, biological sciences, social sciences, and humanities which until now have been forced to use nothing more than statistical techniques, usually correlation-regression analysis, and related techniques such as analysis of variance, and principal component analysis.

We (humanity) now have only a toehold on linearity. Every kind of mathematics that is not linear is nonlinear. That is like saying, we now understand elephants, and have just started our work on understanding nonelephants. The only reason why this state exists is because of our incapacity to do more. It is only now with ubiquitous computers that we can delve into nonlinearity. Even a genius like von Neumann had started using computers to study nonlinear differential equations. There is a need to combine linearity with nonlinearity, and there is a need to have multiplication as a part of the model-making stage of science (see further below).

Like many things in the universe, knowledge is a product of an *intensive* and *extensive*

parameter. In gymnastics, diving, figure skating, the score of the contestant(s) is obtained by multiplying an intensive parameter with an extensive one. The degree-of-difficulty of the routine is multiplied by how well it was performed. It seems odd that an idea that can be understood and practiced by jocks still has escaped the great intellectuals of the 20th century. A philosophy that does not take such things cannot be anything but a sterile replay of the past centuries. It can only be regression not progression. The concept of entropy (from thermodynamics which should really be called thermostatics) is one of the greatest abstractions to come out of the last century if not the greatest and is the only link of physics to biology and biological systems. In it quantities like $p \cdot dV$ and $T \cdot dS$ are energy terms (product of *intensive* and *extensive* variables).

Thus in science, mathematics is the intensity of knowing.Volume of knowledge is a different concept. Trivial pursuit will never be science. Its intensity is near zero. Quiz shows on who played in what TV series will never be science although they can be considered to be a test of (some kind of) knowledge i.e. a low grade knowledge. Today's sterile philosophy has much in common with the kinds of low-grade knowledge above. One cannot reclothe a 2,000 year old idea and expect it to be something it cannot be, ever. Physical sciences lead all sciences because the intensity of knowledge in physics is high. That is because (1) physics is easy (2) physicists use powerful tools. Social science does it in reverse. Their field is immensely complex and difficult and they are working with stone age tools [Hubey,1996]. And this is also easy to explain. It is *triage*. The intellectual energy of society (and this not only includes engineering, biology, mathematics, computer science but does not include sterile philosophy or useless speculation) is directed towards those things that will produce the greatest output and what people seem to want. People want cars, TVs, VCRs, food, entertainment. They don't want enlightenment and they don't want to know what will happen after they die. They don't even want education. *The masses have revolted*. Even if most of the energy of society was directed towards the areas which humans are supposed to strive to understand (i.e. proper study of man is mankind), there would probably be no success soon anyway because the problems are so difficult.

That is why the greatest intellectual tools of humanity (mathematics) are brought to bear on physical things. The problems are easy and solvable and produces tangible results. The order in which things should properly develop was enunciated almost 200 years ago by August Comte.

3. Non-piecewise-linear Triage

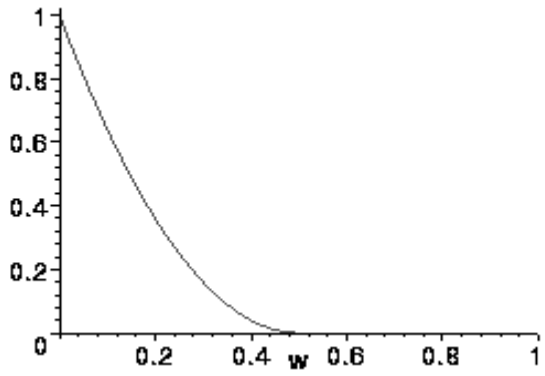

Figure 11: Parabolic for the Nonwounded

$$T_0 = 4(W-0.5) - 4U(W-0.5)(W-0.5)$$

The implication for the parabolic coding is

$$8) \qquad m(W,T)_p = T(-4Z+1) + (1-T)Z - U^{\dagger}Z$$

The corresponding implication for the parabolic is plotted below in Fig. (12)

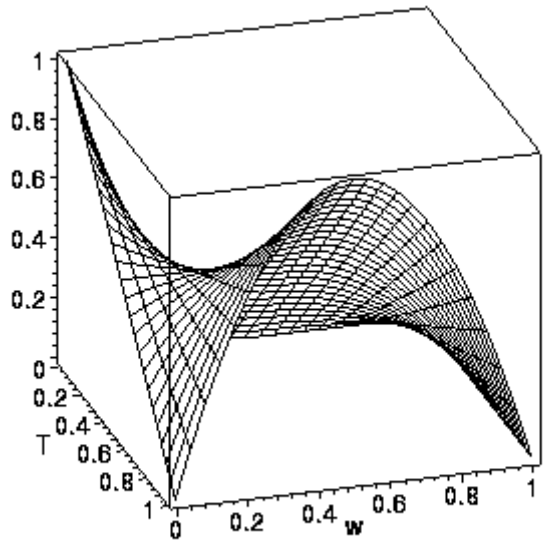

Figure 12: The implication using parabolic valuation

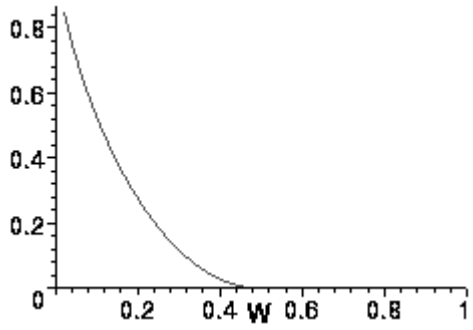

Figure 13: Round Nonwounded

$$T_0 = 1 - T_1 - U^{\dagger}(1 - T_1)$$

9) $$m(W, T)_R = TT_1 + + (1 - T)(1 - T_1) - U^{\dagger}(1 - T_1)$$

for the 'round triage'.

4. Tnorm and Conorm

Even in logic it is difficult to define "truth". The best definition is via the definition of falsehood and is explicit in the definition of implication. $P \Rightarrow Q$ is false only if P is true and Q is false. That is the "counterexample". So a statement is only false if there is a counterexample. Else it is true. Both Popper's vision of science and philosophical logic and hence modern philosophy is based on this. The same goes for science. We look for patterns, shapes, order, structure etc. After having cast around for a while I realized that as above this is dependent on the definition of random (as in random vs deterministic) or chance, etc. The triage implication is, then, a part of the nonlinear (multiplicative) model building part of science since we can derive the t-norm and the t-conorm from the implication. Because of the relationship between the implication, intersection and union given by

10) $$(W \Rightarrow R) \equiv \overline{W} + R \equiv \overline{(W\overline{R})}$$

the t-norm and the conorm for triangular triage can be obtained easily from the implication.

11) $$(W \cap T)(x) = \begin{cases} (1 - (2(1 - T)W + T(1 - 2W))) & W < \frac{1}{2} \\ 1 - 2(1 - T)(1 - W) & otherwise \end{cases}$$

and

12) $$(W \cup T)(x) = = \begin{cases} 2T(1 - W) + (2W - 1)(1 - T) & W < \frac{1}{2} \\ 2TW & otherwise \end{cases}$$

It is easy to obtain $(W \cup T)(x)$ from $m(W, T)_L$ since all we need to do is substitute the complement of W into the implication. Then we can obtain the $(W \cap T)(x)$ by complementing $(W \cup T)(x)$ and then substituting the complements of W and R into it. The same algorithm can be used to derive the t-norm and conorm for the parabolic and the round cases.

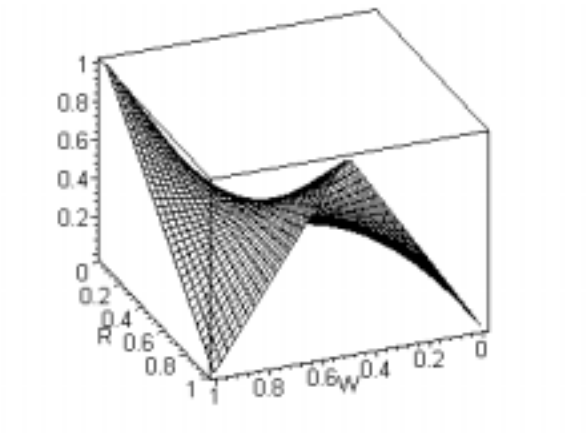

Figure 14: Triangular Conorm

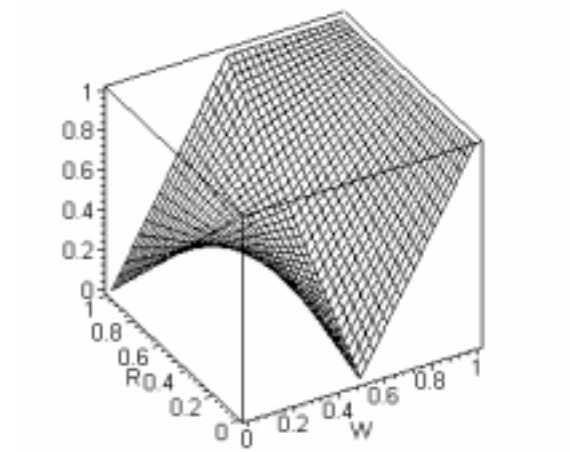

Figure 15: The Triangular t-norm

5. Conclusion

Anyone who looks for answers already has admitted that there are answers. Anyone who starts looking for structure has already admitted to the existence of order, structure etc. A general and common definition of science is in Collins English Dictionary (1991): "The systematic study of the nature and behavior of the material and physical universe, based upon observation, experiment, and measurement, and the formulation of laws to describe these facts in general".

Mathematics is the only thing that studies patterns and structures devoid often of any of these having any models in the real world. That is why it must be the language of every science. However, there are other stages that are involved in natural science investigation. The first is determining whether an isomorphism can be proposed between a putative unit of the construct, and the numbers used to represent that unit. This isomorphism can be proposed on either a theoretical basis, or on the basis of empirical investigation. Often it is easier to create an isomorphism between sets of parameters of the system under study and some mathematical model. This model can then be used to determine tools for making quantitative measurements. Typically in the social and life sciences, there is much stress on types of measurements (called scales of measurement) such as nominal scale, ordinal scale, difference scale and ratio scale. In complex systems it might not be possible to study a particular quantity and create a scale for it. An example from the history of science is the study of heat and temperature. It took a long time before temperature was linked inextricably with the laws of thermodynamics in a clear way. It was not until last century that an absolute temperature scale was not posited and it was based on study of heat and the laws of thermodynamics based on empirical studies. Thus making mathematical models even if ratio scale measurements do not exist is still extremely useful. The extensions to fuzzy logic proposed here are more tools in the bag of the scientist.

References

Hubey, H.M. (1994) *Mathematical and Computational Linguistics*, Mir Domu Tvoemu, Moscow, Russia

Hubey, H.M. (1999) *The Diagonal Infinity: problems of multiple scales*, World Scientific, Singapore.

Hubey, H.M. (2000) *Fuzzy Operations*, 4th World Multiconference on Systematics, Cybernetics and Informatics-SCI2000, July 23-26, 2000, Orlando, Florida.

Hubey, H.M. (1996) *Topology of Thought*, CC AI; The Journal for the Integrated Study of Artificial Intelligence, Cognitive Science, and Applied Epistemology, Vol 13, No.2-3, 225-292.