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CONCEPTS AND COMPLEX SYSTEMS

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TOPOLOGY OF THOUGHT

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Abstract

Now that interdisciplinary activity seems to have started in earnest in the field which concerns the mind/brain, natural/artificial intelligence, machines, and problems of knowledge, we find that communication among the researchers is difficult. If we can identify the commonalities of the concepts used in the physical and social sciences, we can explain and discuss findings in different domains in ways accessible to researchers in the sciences, philosophers and even students. To do so we must be able to find similarities and differences which can best be done in some appropriate perceptual metric space. Hence this paper discusses the various dimensions of the concepts, primarily those that comprise what is perceived as complexity in its most general sense, that are shared by different domains, and proposes perceptual metric spaces appropriate for the task of unifying and integrating scientific fields including epistemology, cognition, and artificial and natural intelligence.

0. Introduction

A famous preacher was asked about the success of his sermons. He replied, "It's easy. First I tell'em what I'm gonna tell'em. Then I tell'em. Then I tell'em what I told'em." This preacher-model of communication is probably fine for 'content-synchronized' sessions as in computer networking [Tanenbaum,1988]. In this book Tanenbaum gives a beautiful analogy of the layers of computer communication and how they related to each other via an example of two philosophers in different countries discussing rabbits. However, discussions of topics such as the role of complexity, chaos, and nonlinearity in epistemology, and/or cognition are based on related topics such as artificial/natural intelligence, and the role of knowledge but there is no common background (or a core set) of knowledge, beliefs or even a set of tools for analysis or discussion. This state of affairs, that is, the set of assumptions, knowledge, a point of view/perspective or even terminology that each researcher brings to the study/discussion of the problems, is a serious barrier to communication. Therefore, before the preacher-model of discussion can be effective, we must have the common ground in the milieu that the communication attempt presumes.

In particular, the disagreements in the field which roughly revolves around the idea of knowledge (information/data processing), and various ways of accumulating, storing, comprehending, organizing, transmitting or otherwise using it, are compounded by the backgrounds of the people dabbling in the field e.g. researchers from psychology, philosophy, computer science, engineering, physics, and bioscience, specifically neurophysics. Demonstrating that there are standard and common methods of analysis used in various fields would facilitate clear discussions of what new additions chaos theory, catastrophe theory, the study of nonlinear phenomena, and theory of complexity bring to dissolve some of the timeless and perplexing questions of the social sciences, biological sciences and philosophy, particularly philosophy of science, and epistemology. Therefore the paper's outline is as follows:

I.a: The Standard Ontology of Epistemology: Where are we?
The state of the art, and of the knowledge workers. Who are they? Is epistemology still philosophy? What tools are in use and by whom? Why these specific tools?

I.b: Various Philosophical Isms: How did we get here? (A Simple Taxonomy)
To avoid "re-discovering the wheel" there's a short discussion of the basics of philosophy of science-- the standard Baconian, Kuhnian, Feyerabend views, which are, in effect, the standard ruminations of philosophers. Also in this section are general questions about what is philosophy, and complexity? Is philosophy a watered-down science? Is it science for the masses? Is it a methodology superior to science?

II: Fundamental Dimensions of Classical Tools: Roots of Miscommunication
A unified and integrated look at the human problem solving tools in use and applications to various problems is in this section. The usefulness of such tools according to the complexity of the problem, graininess of the phenomena we observe and the precision with which we can wield such tools are discussed . The two main schools of thought according to the standard paradigms, according to the background beliefs and tools wielded, and the advantages and disadvantages of each are examined.

III. Science of Philosophy
There is a philosophy of everything; is there a science in/of philosophy? What are the main tools of thought? A classification in 3-dimensional (3D) space for simplicity, a reasonably modest start, is given in this section. Then the general and recurring ideas in all sciences are given

IV. Concepts and Complexity
Complexity in computer science essentially refers to some kind of a magnitude/size/scale. Most of the time it refers to time-complexity since it is more important; we cannot make more time but we can make more space resources available. But informally complexity means more than the computer science definition. Here I try to produce a more general and generic concept of complexity by examples, and by using the previously developed concepts and try to slant it so that people from both sides of the great divide (please see section II) can see it in terms of and relate it to their own fields. Furthermore a taxonomy along the lines of binary trees is given. Then in keeping up with what was done earlier, I produce the metric space based on the topology of dichotomous taxonomy. Examples from fields which are assumed to be diametrically opposed, science and art are given; philosophy is included to make it a three-way comparison.

Appendix 0: Patterns
A categorization of various mathematical tools used in various fields in similar ways and their analogues in verbal mathematics.

Appendix I: Direction and Entropy in Intelligent Macroscopic Systems
The macroscopic example in the appendix demonstrates the appropriateness of the thermodynamic/systems point of view in modeling/examining "complex" systems such as human societies/cultures. It deals with the system at a macroscopic level over large time scales in which one can see deterministic law-like behavior.

Ia. The Standard Ontology of Epistemology

One of the highly self-evident received wisdoms of the latter part of this century is that we have entered a new age, christened in various books and journals as the *communication era*, the *information age*, the *post-industrial age*, the *communications age* and so on. What is behind all of this, of course, is well known to be the now ubiquitous digital computer, and the digital electronics revolution. Although it is agreed by all that the computer now allows us to finally do for our mind what mechanical machines allowed us to do for our physical bodies (that is, magnify or amplify our natural genetically endowed capabilities), whether the computer is an electronic brain or an electronic brawn is still controversial. Another controversy revolves around the [alleged] impossibility of the separation of knowledge (and hence its study, epistemology) from the knower (the human mind/brain), therefore any discussion of one naturally leads to the other. However, some of the intrusions of the modern age upon this long-held view is that not only might it be possible to define knowledge separately and apart from the [human] mind/brain that possesses it, but it also might be possible for machines to have minds. Probably the primary reason why the battle lines are drawn against the possibility of machines having intelligence is that it implies that it is possible that we are also machines, a possibility fraught with emotion at the moment however not really threatening if examined in a different light. The arguments that rage about these theses are partially semantic, and a better comprehension of the meanings we normally attach to common words such as machine, mechanical, deterministic, random would go a long way in stopping useless arguments. Other arguments impinge upon basic concepts (some of which are still not clearly defined or understood) which could change in meaning as a result of some of the inventions of recent history. For example, are we becoming Hegelians? In other words, is "emergent property" a euphemism for "quantity leads to quality"? If so, then is there really a difference between electronic brawn and electronic brain? If the artificial neural networks (ANNs) get bigger and bigger will we start to see behavior which we will judge to be "qualitatively different" than the simple behaviors which we can already see and attribute to small ANNs? Do we know how to tell 'quality' from 'quantity' ? Are there quantifiable qualities? Is it really the case that human behavior is 'qualitatively different' than, say, that of chimps? Or is it the case that we seem to have more of 'it' (whatever it is) which we see as a 'qualitative difference'? Should we chuck this time-honored tradition of *quality* vs *quantity* and replace them with something more mathematical or better defined, such as extensive vs intensive?

What then of mathematics? Which branches of it are more suitable for new [social and life] sciences? Probably the most highly *stylized fact* of mathematics this century is the concept, a set is the most important and general concept from which everything flows. One can call this a dogma from which almost no one has been immune, the notable exception being Hellman (1989) who challenges the view not only that set theory is the foundation of mathematics but even the view that mathematics has to have foundations. Does mathematics have to have foundations, like a hierarchical tree structures of communist countries, the Catholic Church, the military, the Linnean tree of life, directory trees of computers, or does the enormous complexity of life necessitate that we take a lesson from real life (such as capitalist systems where there are many places where the *buck stops*) and from computer networking (where the protocols are for decentralized control among peers). There is something natural and familiar about this *boxes-inside-boxes* metaphor of organization (i.e. apartment house, apartments, rooms, closets, drawers, jewelry boxes, etc.) Hence the ideas of self-similarity, repetition, iteration, recursiveness have always been with us. It seems that parallel processing is more like the operation of the world in which interdependent but autonomous entities roam. So too, now with peer-to-peer protocols in networks, massively parallel computers, and tightly-coupled networks we are once again creating a virtual world in the image of the real world. Instead of asking very difficult questions in areas which are fuzzily defined and understood, we might be better off asking questions in areas which are more precisely defined first, and then think of their analogues in the real world. What kinds of ideas or concepts do we use in studying natural phenomena aside from the most basic one of sets? We can immediately recognize the idea of analogy since we've been using it all along, and there are more. We can quickly see and list some other common ones with which most people are familiar; static vs dynamic, parallel vs serial, union-intersection, analogical reasoning, inductive learning, sameness vs difference. These seem useful for taxonomic purposes, but are there more? What of the black box model? There is no shortage of applications of the concept (Naylor & Sell, 1982). Are the social sciences that much different, that no mathematics used successfully in physics can also be employed in this area? The topic of whether the social sciences can learn something from physics ranges from what has been dubbed *physics envy* to outright rejection of the methodology as inappropriate for life sciences. The usefulness of mathematics is usually not openly questioned, certainly not by most philosophers since it is deemed neutral with respect to application, but the attacks are indirect since determinism is opposed (mathematical functions, differential/difference equations etc.), and so is randomness (probability theory), and despite this most refuse to use anything but bivalent logic. It seems there is an element of irrationality in this view that is difficult to ignore. However, since we know that mathematics developed in tandem with physics, except for some recent impossibility theorems, and those which depend explicitly and implicitly on computation, it is possible that mathematics is not yet powerful enough to capture the complexities of social phenomena. But what makes mathematics unsuitable (if so), and what alternative methods have philosophers developed to tackle the great unsolved problems of science (or of the universe)? Is there some unified way in which we can view scientific knowledge as it developed during the past few thousand years? Are there some tools of science that seem to work that can be visualized according to some taxonomy or structure? The first thing to come to mind will probably be mathematics, and we can organize mathematics the traditional ways in which it has been organized. However, an organization of a vast field that is optimized for a semester-at-a-time learning is not necessarily the best method of organization for every purpose.

In that vein, we can make a very modest effort at categorization of concepts from mathematics, philosophy, and examples from physics, engineering and computer science which are more suitable for discussion as they pertain to being tools appropriate for the problems for which they are used. This seems an especially opportune time to do so since, once again, the old brain-mind problem has been surfacing under the guise of seemingly different concepts, and especially during this time of great interdisciplinary progress in this field of epistemology, knowledge/information, Devlin (1991); mind-brain, Davies & Stone (1995), Chappel (1981), Ornstein (1986); neurophysics Churchland (1995); cognition Posner (1993), Newell (1990), Lycan (1990); mathematics, and computer science, Churchland & Sejnowski (1993); consciousness, Dennett (1991), Damasio (1994), Jackendoff (1987); especially the area of machine (artificial) or natural intelligence Boden (1990), Cherniak (1986), Gardner (1983), Hubey (1996). The first thing we note is that classification/categorization is dependent on the criteria we choose for the taxonomy. We should necessarily go further than simply creating sets, and bequeth some structure to this categorization/classification space. Since we are dealing with high level abstraction, a trade-off between precision and/or accuracy and simplicity is expected. Therefore the *space* has to be of low dimensionality for obvious reasons; one is that we cannot readily visualize more than three dimensions (four if we include time). The other is simply that it's next to impossible to draw more than 3 dimensions on a two-dimensional communication channel such as paper, however it will be shown how the dimensionality can be increased while keeping the topology invariant. One thing that we note, is that some ideas/concepts show up in what ostensibly should be independent/orthogonal dimensions. We're not used to thinking about dimensions of phenomena being dependent upon each other, however it will be shown below that there is a very good reason for this trade-off. These problems need to be discussed in detail for clarification and hence we can see that study goes from analysis to synthesis (or in more modern terminology, *bottom-up*) and *top-down* (synthesis follows analysis). For given problems we can start with a shallow breadth-first search followed by in-depth searching of promising avenues, and backtracking. All of these methods are instinctively followed in discourse, books, and journal papers, indeed, throughout our educational system, since we really learn much the same things not only at deeper and more detailed but also higher levels using more powerful methods (mathematics). However in this case, taxonomy itself is too important to ignore, and to this we must first turn.

Ib. Various Philosophical Isms: Taxonomy - How Did We Get Here?

The first stage in the development of science seems to be classification according to some criteria. Many "sciences" never get beyond this taxonomy stage, certainly not to stages where measurement plays an important role. Of course, measurement immediately implies a dimension and a distance metric of sorts, and hence probably a metric space. The easiest (and crudest) scheme in taxonomy/classification is to use dichotomy i.e. a binary measure. And this binary measurement, say {0,1} (the standard measure of information and hence knowledge itself) is enough to create a metric space of its own. For example, suppose we're given some objects, which are only black or white, only pyramids or tetrahedra, and either made of iron or aluminum. We can immediately see that we can use three criteria {color, shape, material}. Normally a taxonomic structure is given as a tree, the most famous one probably being the taxonomic trees of biology. At the first level we can select one criteria (say color), then a second level, (say shape), and then the last one, material. So we now have a tree as shown in Figure I. We note is that this is not the typical binary tree of computer science. The nodes at each level are labeled so that the numbering

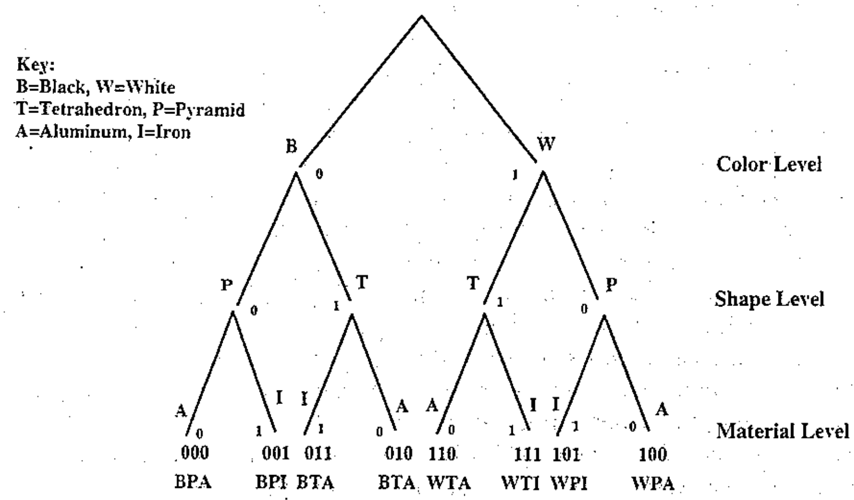


Figure I: A Taxonomic Tree

scheme of the two halves at each level are reflections of each other. Of course, this is nothing but the Gray code, and immediately we see that there's a distance associated (Hamming distance) with the tree as and it can be seen in higher dimensions as in Figure II.

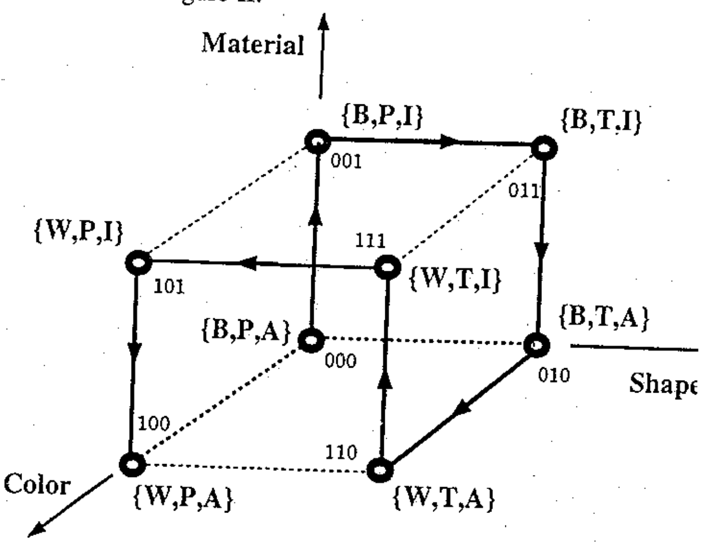


Figure II: Binary Taxonomic Hypercube
(A Hamiltonian Traversal of the Nodes Is Shown)

It's obvious that the idea of distinctive features in linguistics also has the same metric space associated with it (Hubey, 1994), and the same idea of *shared features* can be found (and used) in biology and paleontology to create a simple metric space for different species of animals and plants (Hubey, 1995). Categorization is based on such shared properties, although used heavily by the biological sciences, Eldredge & Craft (1980). In the binary cube (Fig. II) we see that each node is one bit away from the other nodes to which an edge exists. Hence, this binary cube displays the Hamming distances based on the reflected code. Furthermore, there is a Hamiltonian path that reaches each node only once, and in such a way that each node has the distance one bit from the previous one on the path, and one bit from the next node. Therefore the path, shown in Fig. II, is one particular way to serialize the 3D picture so that information can be communicated over a serial channel such as speech or writing. Furthermore we can see in the plot that this path is only one of many since we can start at any node, and have two choices at each node.

We already know that there is no reason why we should have picked color to be the most important criteria for classification purposes although in particular situations it could make sense. We selected one particular permutation, that is {C,S,M}, whereas we also have other permutations available such as {C,M,S}, {S,M,C}, {S,C,M}, {M,C,S}, or {M,S,C}. It turns out that we can represent these in 3 dimensional space as vectors, and that the various views/perspectives can be shown to be a group, and that the various views are isomorphic to one another Hubey (1994). We can extend this idea to infinite dimensions Hubey (1997). If we do not use the these Hamiltonian paths we can simply use zigzag traversals using the standard binary counting to organize the hierarchy of discussion since by its nature such discussions must be serial and not parallel, Hubey (1997). We can also change the space of measurement which is discrete and binary {0,1} to a scale of normalized intensities [0,1] in some circumstances by finagling around with the dimensions. For example, we can change the dimension for material from aluminum to metal, or if other materials are included on the basis of some mechanical property such as Young's modulus or hardness, or even on the basis of some weighted measurement of mechanical properties such as Young's modulus, Lame parameters, hardness, brittleness, ductility, etc. On this basis we can look at this taxonomy yet another way. In Figure III the dual of the graph (the cube) has been drawn, in which the edges

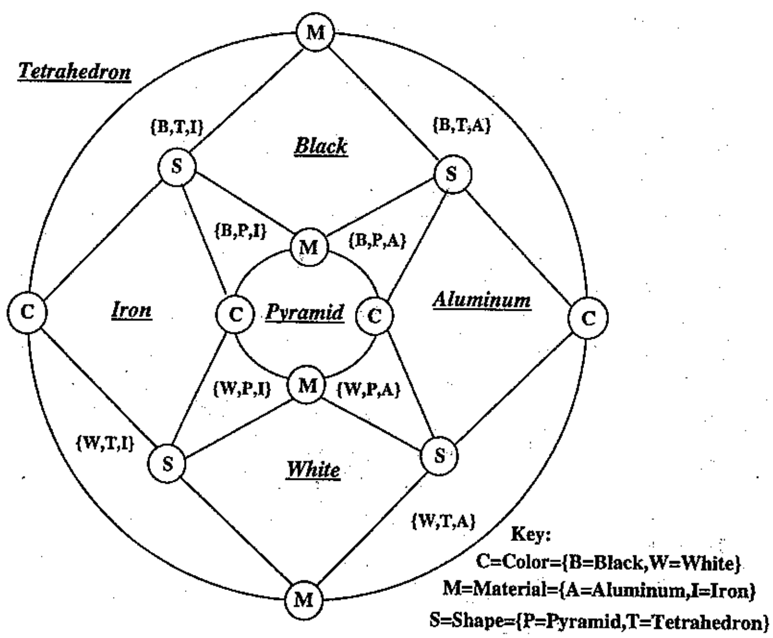


Figure III: Dual Space of the Taxonomic HyperCube

(The outside (i.e. tetrahedron) is another 4-faced area on the surface of a sphere)

become nodes, and the graph is planar. The faces of the cube are still faces of the dual (still four-sided), but the nodes of the cube now become edges (with duplicates) or three-sided faces. For example we can see in the graph that the triangular face {B,T,A} borders on three four-sided faces (Black, Tetrahedron, and Aluminum). We note in passing that the dual graph is easier to visualize if wrapped on a sphere. Therefore the face labeled "Pyramid" is opposite the face labeled "Tetrahedron". Similarly for Black/White and Iron/Aluminum. The nodes labeled M split the surface of the sphere into two sets (Iron/Aluminum). We can see that a line running through these nodes will be a great circle. Ditto for the C and S node sets, except that as drawn the lines connecting the S set can be seen to be roughly circular in this view but the others will have to be imagined wrapped on a sphere.

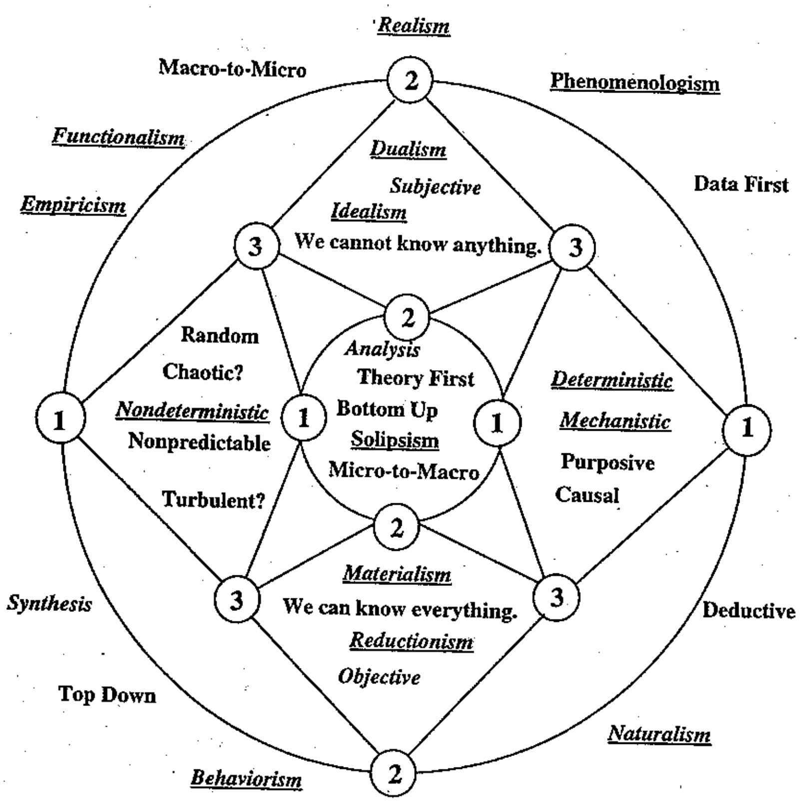


Figure IV: Dual Property Graphs - Very Simple Representation of Philosophy/Epistemology

If we try to produce metric spaces or geometries for thought/philosophy we are immediately confronted with a veritable mountain of ideas, dichotomies, view points and elaborations such as idealism, dualism, mechanism, determinism, behaviorism, idealism, objectivism, phenomenologism, naturalism, solipsism, atomism, reductionism, materialism, dialectics without even getting into the present day rejuvenations such as behaviorism of all flavors, reductive materialism (identity theory), functionalism, eliminative materialism, idealism, methodological behaviorism, methodological solipsism, (see for example, Churchland (1979, 1994), Millikan (1984), Ihde (1986), Skinner (1965, 1972, 1974), Lycan (1990), Boden (1990), Habermas (1988), Lyotard (1991)). Only a toy domain, with ideas taken from the distant past, will be considered as an example. The dimensions of such a space must be comprised of the answers to the standard questions of philosophy. What kinds of objects we deal with? Are what we perceive real or are they only mere figments of our imagination; in other words, does the real world exist independently of our observation, or is it only there when we perceive it. Is there an objective reality or is everything subjective? On the whole, is the universe deterministic/causal/mechanistic or is it random/chaotic? How do we observe nature/reality/phenomena? Do we need a theory to observe (measure) or do we theorize after measurement (observation)? What's a theory? Is it simply a mathematical model or are verbal descriptions also theories (i.e. what is science)? These questions are asked not because we should start questioning from here as it would resemble eating pre-chewed food, rather because these are the end points of different systems of thought which have existed throughout centuries in one form or another, and probably used for taxonomic purposes even today, although probably better taxonomies (for different purposes) are available. Therefore, we can intuitively see that the ideas which have been tossed about for centuries must be aggregated (grouped multiplicatively — to be explained in more detail later) to reduce the dimensionality of the problem. One of the most obvious problems is that the three putative dimensions of this taxonomy (as shown in Table I) aside from being woefully inadequate to faithfully represent the details, are all based on something else, that of scale or size. We can see, for example, that the world view as our basic and fundamental starting point is available to us only via our naked senses, and our biological capabilities limit the scales at which we can sense phenomena.The bandwidth of visible light is a minute portion of the electromagnetic spectrum, the range of hearing is but a fraction of the acoustic frequencies available in nature, the resolving power of our eyes is not sufficient to see microbes let alone atoms, and our physical strength allows us to handle only light objects. Since we couldn't fly even something low-tech as map making (bird's eye view) had to be a roundabout affair. Our most primary intuitions, and common-sensical views (including folk psychology) are limited by scales. The

1. Nature of Reality (if it exists!): Objects of the Universe and our Perspective

Apparent (Virtual)	*Real (Actual)*
Dualism (various kinds)	*Monism/Materialism/Reductionism*
Idealism	*Materialism/Physicalism/Reductionism*
Reality and Appearance are not same	*Appearance (observation) is reality (Realism)*
Intrinsic essences beyond physicalism	*Purely descriptive*
Subjective/(Internal-to-External)/Introspective	*Objective (External-to-Internal)*
We cannot know anything.	*We can know everything.*

2. Nature of Interactions (Dynamics / Mechanisms)

Deterministic	Non-deterministic
Causal	Noncausal
Mechanistic	
Purposive	*Chaotic(?)*
Predictive, Predictable	*Non-predictable (Free Will?)*
Laminar	*Turbulent*

3. Methodology: Ordering World View (How shall we proceed?)

Micro-to-Macro	*Macro-to-Macro*
Analysis/Bottom-Up	*Synthesis/Top-Down*
One-to-many generalization / inductive	*Many-to-one specialization / deductive*
Solipsism	*Naturalism/Empiricism/Phenomenologism*
	Behaviorism/Functionalism
Theory determines what can be measured.	*There is a reality independent of observer.*
Theory first.	*Data First.*
We are all same (equal).	*We are all unique (different).*

Table 1: A Very Simple Philosophical Taxonomy

nature of interactions of the objects of the natural universe and indeed the *natures of the objects* themselves (as we perceive them) are determined by the scale of the phenomena, and the capability of our transducers which allow us to observe them. Even the whole idea of duality, that appearances are not reality etc. can be thought of as a problem of scales, since something that looks contiguous or whole (i.e. an apple) is really composed of many smaller parts, molecules, atoms, etc. For example, the idea previously alluded to, namely that it is the theory that decides what can be observed/measured (due to Comte and popularized by others) vs. the old scientific view that data is collected and then a theory is proposed is such a problem of scales in physics. Data could be collected for phenomena which we could not influence, (such as the motion of the planets) or phenomena which we could observe as a result of carefully controlled experiments in a laboratory (and the results of which did not need any more than observation with the naked senses) or even using instruments whose operations were very close to our common-sensical views, so that questioning what the instrument measured did not even occur to the experimenters (or even the philosophers of the time period).

The next example comes from social sciences; we have now gone from the *society-made-me-do-it* view to the now fashionable *you-have-a-choice* paradigm. The fact is, both are true and related to the problem of determinism vs. free will (as well as being a problem of scale). Society's (and our immediate surrounding's) effects occur over a long period of time but our actions (such as pulling a trigger) are of a momentary nature.We are what society made us and we are also unique (Hubey, 1993). Indeed, even the teaching of philosophy is dependent on scales. The historical method is not only inadequate but also incorrect, in the sense that attempts to extrapolate to later philosophies (which were born of the need to reconcile the old modes of thought with new scientific discoveries and understanding of the universe) to obtain the newer ones, is, in some ways, like being forced to give lectures in chemistry by extrapolating from the ancient Greek elements such as water, air, fire, etc. The proper way to indulge in such teaching is to start with modern chemistry and explain the misguided alchemy and Greek fundamental elements in terms of modern ideas.

What is philosophy and what is complexity? We have only intuitive understanding of concepts such as *complex, difficult, abstract, object*, and we have to start there and then bootstrap, as is done in science and as seen in the history and prehistory of humanity. We normally associate complexity with difficulty, and philosophy with abstractness. Hence intuitively we can associate the concepts of difficulty, abstractness, haziness, with complexity. One kind of difficulty is due to novelty, something so unfamiliar that we can't even describe it. Another kind is due to taking a long time to accomplish a task, and yet a third is due to haziness, or abstractness of concepts which is probably the common man's view of philosophy. The problem as in all facets of things studied by humans is to reduce the complexity to make the *thing* comprehensible. Therefore the natural question is how do we reduce complexity of things. The basic idea of splitting pieces off the larger object to study its nature is used all the time, and goes under various names/dichotomies such as analysis/synthesis, modularization, and decoupling, but we will take a slightly different approach here to examine what we mean by modularization, or analysis or how we decouple phenomena. As in the case of the divisions or views of the simple example given in the above section, we can note that some of the ideas can be grouped around dimensions that we can obtain via *multiplication*. There are two reasons for this. One reason is that it is a remarkable fact that almost all of the simple laws of physics are multiplicative, for example, $i=GV$ (where i=current, V=voltage, and G=conductance) and $a=F\lambda$ (where $\lambda=1/m$=*lightness*). In both cases, some flow or motion is proportional to a product of some kind of a force applied and some property of the object appropriate to the situation. The second reason is really an elaboration of the first in logical/philosophical terms since, multiplication corresponds to a logical-AND (whereas additive models are logical-OR models, which assume gross substitutability among the variables, such as in the nature vs. nurture debates, Hubey (1996)). We see that almost all psychometric tests should be multiplicative models (or mixtures in which case the present linear regression-correlation tests fail) or as approximations to multiplicative ones (using only the linear terms of Taylor expansions). Obviously nonlinearity, by definition, involves multiplicative models.

It is now reasonable to expect that we can create a 3D space for ideas/thoughts of the previous section using the dichotomous method explained in the earlier section, and that some of the philosophical viewpoints/perspectives which are hybrids can be cast into this space by fractionalizing them by changing the $\{0,1\}$ space into $[0,1]$ spaces as in fuzzy logic. As simple as it is, it would still seem to be an improvement over much of what exists presently since most of it is really expressed in 2D media such as writing (which phenomenology takes so seriously as to take reading as fundamental in communication of ideas). We would not be too from truth to claim that since language is serial, much communication is single-dimensional. In this sense we already have an analogy to the concept in chaos in which we can construct the multidimensional attractor from a single time series. In that sense, although language is 1D, the construction of meaning in the brain/mind is multidimensional. The real question is if there are some common concepts which can be used as analogues in various ways of world making. Surely, we want more than sets which have almost no structure.

II. The Fundamental Dimensions of Classical Tools : Roots of Miscommunication

The first problem we face when we wonder how knowledge is possible, is that separation of knowledge from the knower is difficult. If it were not so, then knowledge has already existed in the universe since its creation and we simply discover it, much the same way as we accidentally run into new continents. The corollary of the above is that it's difficult to separate knowledge from intelligence since problem-solving is also knowledge. Traditionally animals were not thought to be endowed with intelligence as humans but only instinct, hence animals could not be taught but only trained. Even now, intelligence is usually means human level/type intelligence, although there are exceptions, Walker (1983). One can say that the impetus for separating knowledge from the knower (always human) came with the possibility of digital machines and hence artificial intelligence (AI). It became obvious immediately to researchers in AI that an inference engine is insufficient to create intelligence or solve problems; a knowledge database is also needed. Here, we take two steps seemingly in two opposing directions; while allowing for the possibility of animals and machines to have intelligence, we also allow for the possibility of separating knowledge from the knower. We can go further and posit that knowledge comes in intensities like heat and that mathematical knowledge (model) is of the highest intensity, which, of course, doesn't guarantee *accuracy*, so that in this sense, the intensity is that of *precision*. The words, *reliability* and *validity* in the social sciences are the analogues of *precision* and *accuracy*, so that the analogy of mathematics as an instrument for epistemology (in the broadest sense since it now includes or should include all forms of knowledge) is simply that it is precise/reliable but not necessarily accurate. The latest additions to our mathematical toolkit will be discussed later but there are plenty of excellent references to *verbal mathematics* of which every educated person should be informed, such as Kline (1973), Aspray & Kitcher (1988), Benacerraf & Putnam (1983), Aleksandrov, Kolmogorov & Lavrentev (1963), and others with more philosophical overtones such as Putnam (1975), Korner (1960), Hellman (1989), and even others which touch directly upon epistemology from the point of view of logic, or probability theory; Armstrong (1973), Devlin (1991), Mackie (1973), Poundstone (1962), Ayer (1972), Swinburne (1973).

If we take the mathematization of knowledge as science as one of the fundamental postulates or tenets of the new view of what comprises science, we can create a space for the discussion and study of natural phenomena (which naturally includes the mind and knowledge). The purpose of this space to show the tools that we use in analysis and synthesis in our studies (see figure V). One of the

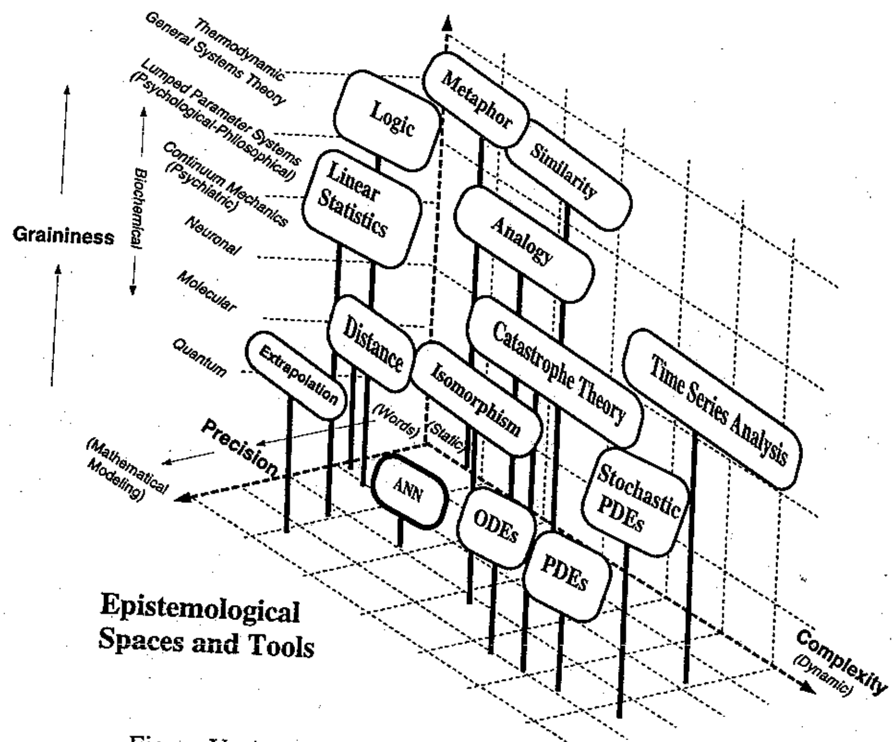


Figure V: Appropriate Tools for Appropriate Phenomena
(The fields actually occupy volumes so that only approximate placements of representative instantiatons have been shown. Since it is still difficult to separate knowledge from the knower, the graininess has an anthropomorphic flavor to it; hence the given levels. The psychiatric level can be thought of as a combination of psychological and chemical, and it could be a region that extends to include the psychological)

biggest hurdles to communication in interdisciplinary phenomena is the nonexistence of the common core of terminology or tools. Even in such a field as communication, as recently as a few years ago, the integration of old telephony, computer communication, and the media types via the heavy use of digital communication technology (e.g. ISO LAN, WAN standards such as ISDN) made communication difficult for people hailing from the computer, telephony, and the broadcast fields. Exactly because (the new) *epistemology* (and all that it entails such as intelligence, scientific knowledge, consciousness) is interdisciplinary, there are several tribal languages in use. In some cases, just because the phenomena is too complex, it is thought by one tribe that no kind of mathematics will do, usually by those who don't know any. On the other side of the fence , we have people, (for example Comte as early as during the last century), who ask why people who can't even solve easy problems (usually in the hard sciences) claim to have solved hard ones (usually in the soft, social sciences). The construction of a toy model (like prototypes in wind tunnels) is thought by some to exemplify a kind of a proof by demonstration that the method will work. The idea, in spirit, is similar to the use of ideal gas models. But for others, the arguments are over whether the toy models (prototypes) are so far removed from reality that the prototype and the reality are decoupled. This happened in the past, for example, with the Navier-Stokes (NS) equations for fluid flow and the hydrodynamic equations which engineers used; the NS equations could not be solved except for the simplest cases and they did not seem to have any connection with observation until Prandtl started to show during the early part of this century that what was missing was the special things happening at the boundary layer. It could be argued that this is a perfect model for catastrophe theory (which in turn might be thought of as a mathematization of Kuhn's ideas (1962) about scientific revolutions). We've seen similar things happen in the theoretical equations in biology and economics. It would certainly do no harm to have some idealized toy models in the hard social sciences, like the ideal gas equations still used today to elucidate difficult concepts in thermodynamics although the only gas that the ideal gas equations even approximate is gas at zero pressure (i.e. no gas, at all). Similarly, solutions of Navier-Stokes equations for simple cases, with zero viscosity (i.e. *dry water*) are still useful. Those who scoff at simple models in AI to draw far-reaching conclusions are not taking the history of science seriously. Yet another problem that plagues the discussions in the mind field is the use of arguments from a particular layer (a given level of graininess in Figure V) to extrapolate several layers away. Works of introspective philosopher-psychologists such as Freud, James, Jung, Adler, and Reich, all belong to upper levels, one might say, the cognitive-consciousness-awareness levels in which our thought processes take place in words, and there are many levels in between these and neuron firings due incoming stimuli. In between the input neurons and the output ones, certainly one must expect some high level (and maybe even Freud's subconscious) processes, and even multi-level (e.g. transaction analysis) and multi-personality types of analysis and syntheses. Similar problems crop up in economics when the *real/actual systems* of the opposing camp are compared to *virtual/theoretical systems* of one's choice. Searle's [in]famous Chinese Room thought experiment is of this type and has been clearly torn to shreds at the same level by the Churchlands in their article on the electromagnetically illuminated room, parodying Searle's syntactic, logico-philoso-linguistic type arguments. Arguments of this type are essentially about meanings of words at very high levels in the hierarchy. Indeed philosophy, and linguistics are also very high level phenomena and are closely related to folk psychology (Davis & Stone, 1995, Schultz & Schultz, 1994).

What is really important that can be illustrated with these examples is the description of the tool box, with which we can construct or repair theories of various kinds in the sciences. To a zeroth level of approximation (and approximation is very implicit in logic since it deals with phenomena at very high levels), we can identify at least two schools of thought that collide continuously and repeatedly;

a) physics-mathematics-materialism (PMM -often identified with "hard science")
b) philosophico-linguistic-logical (PLL-often identified as "soft science")

Engineering, computer science and technocrats are naturally identified with PMM while much of psychology is identified with PLL, except perhaps for behaviorists and statistical (clinical) psychologists. What is bringing these together now is the interdisciplinary study of *intelligence,* whether of the real/natural (human and animal) kind, or the artificial (machine) kind, and the associated concepts such as consciousness, knowledge, science, information; with biology now playing the transition zone probably eventually the ultimate arbiter empirically. Biosciences are a part of the "hard sciences" as far as measurements with instrumentation, and objective descriptions but with mathematics seemingly borrowed hodgepodge from various branches and computer science, Eccles (1988), McGaugh et al (1990) while much of it is still descriptive/taxonomic and hence belongs to the soft sciences. The paradigm of the mind is now essentially computational (including connectionism) because the digital computer is the de facto standard of high technology and historically the best technology of the day was used as a metaphor to describe the mind. If both of these approaches (i.e. PMM vs PLL) are scientific (and that can be argued), and if both of these approaches are mathematical (which can also be argued) the dichotomy above is the essential one. We need more than logic, and even statistics. If we want to be able to describe simultaneous effects (especially if we want dynamics) we have to turn to differential (difference) equations(DEs). All the ingredients necessary for descriptions of very complex phenomena can be found in differential equations; we can inject stochasticity, or even account for the famous *ceteris paribus* thought experiments via partial partial differential equations (PDEs). Perturbation solutions/expansions of DEs have the scaling built into them, (Nayfeh, 1985). In this sense fluid dynamics provides the ideal metaphor since the phenomena can be stochastic, is described by nonlinear PDEs. It is more telling that dynamic stochastic process descriptions via the Fokker-Planck-Kolmogorov methods produce equations of this type, Gardiner (1983), Jazwinski

(1970), Soong (1973). The new paradigm for turbulence, namely *chaos*, (see for example, Ott (1993), Sprott (1993), Strogatz (1994), Kaplan & Glasser (1995) or Jackson (1991)) also has all the necessary ingredients to model such complex phenomena since it is *deterministic* and randomness does not have to be "artificially" (additively or multiplicatively) injected into the equations as it is done in stochastic differential equations.

It's obvious that although we don't know enough about humans to be able to clearly take into account their interactions with such equations, although their interactions do comprise what we mean by the word "society". In that sense, we already have very general mathematical models and metaphors for society. As an example of the usefulness of such mathematical models as metaphors, consider the new social fashion in the USA which is "*you have a choice*". In the past, it was "you have no choice because society forces itself to make you what you are." This is also a scale problem. Surely society does affect individuals, it always did, and does now. The analogy here is to thermodynamics; a single molecule does not have much of an effect on the whole ensemble of molecules, but all together they do comprise the ensemble. So what is done, is that mentally a single (representative) molecule is conceptually separated from the rest. All the rest of the ensemble is lumped together as a "heat bath". Then we can study the effects (theoretically, of course, i.e. with equations) of the "heat bath" (the rest of the molecules) on the single (individual, and representative) molecule (Owen, 1984; Haase, 1969; Prigogine & Stenger, 1984). An example of the use of macroscopic thermodynamics to model social phenomena of the role of knowledge in society can be seen in Appendix I.

III. Science of Philosophy

It seems to be a tradition that philosophers are the judges and purveyors of all that is human. From this perspective comes much of the criticism of science and scientists. Various conclusions as to what science is and how it is done, have been reached; for example, (determinist and positivist) Comte (Andreski, 1974), (falsificationist) Popper (1968, 1973, 1983), (there-is-no-scientific-method) Feyerabend (1981, 1987, 1988), (dynamic/revolutionary) Kuhn (1962), and more complete and process oriented views of Lakatos (1970). Among these are those who tried to depart from standard logic towards probabilistic reasoning to give weight to confirmationism such as Swinburne (1973), Mackie (1973), and Ayer (1972), or Carnap (1962, 1967, 1971). Being everything to all things philosophy like a chameleon continues to thrive in the most inhospitable climates, even today. The time might be ripe to ask the question of the existence of *Science* *of/in Philosophy*. We don't have to follow the traditional historical discussion of various philosophies but can rather consider fundamental ideas on the basis of and from the viewpoint of the tools of science. As the various ideas are presented it will be shown that there is a recursivity and feedback, almost like consciousness of consciousness, since the ideas themselves are ordered in similar ways to the discussions which follow. In another analogy we see similarity at different scales, e.g. fractallized foundations.

IIIi. Space (Structure, Dimension, Object, Distance):

We tend always to start with *objects* whether they are of the (abstract) *mathematical* kind, the real kind or the *conceptual* kind (yet not mathematical), and they all exist in some space. Of course, from the *point of view* of perception, knowledge, intelligence, and/or epistemology this is not necessarily *ground zero*, especially if we start with mathematical objects but it will have to do since these are things we can grasp without having any formal training, indeed all animals can, so that in some sense they must be basic. We can all intuitively recognize objects without thinking too much about which properties they have that makes them what they are. Even *set theory* is based on such aggregations without a clear algorithm for recognizing the members of sets. It is only recently with the study of *fuzzy logic*, that the criteria for membership and recognition of such as become more clearly explained. Here we immediately run into the problem of *property/attribute*, that which allegedly makes it possible for us to immediately *classify* and *name* objects. We see that our *taxonomic* capabilities are very well formed as far as everyday objects are concerned. It's worth noting that in engineering (and physics) we use an *inversion* of this form in that we are more interested in objects (or best *representatives* of such objects) as *tensors* of properties. Thus a piece of steel alloy is identified by a vector of properties such as its Young's modulus, Lame parameters, ductility, hardness (on the Rockwell or Brinell scale), electrical conductivity, proportions of other elements such as carbon, nickel, its geometric shape, etc. If we *normalize* the properties in the range [0,1] we can see that we can extend these ideas to include other objects such as cats, apples, trees, or chairs in terms of *fuzzy sets* and fuzzy logic although we are not yet sure that we can *measure* such properties of every dimension of an object the same way as we can measure physical properties. We can always imagine that we can measure *applehood* or *doghood* in terms of definite *ranges* in size, weight, chemical composition, surface color, texture, etc. to various degrees of *accuracy* and *precision*. We can also see that we can never see apple or dog since they are sets and we never see sets/classes but rather their *instantiations* or representatives. This distinction is very basic in phonology since a *phoneme* is essentially a set of *phones*; we never hear a phoneme, only phones. It is this idea of representativeness that fuzzy logic can use to produce *approximate reasoning* for natural *languages*, and the idea of instantiation is now the *paradigm* for *computer* programming since it is now in the *object-oriented programming* stage. Hence instead of considering sets of objects, we consider a specific object, which presently may be considered to be de facto standard in both computer science and in logic via fuzzy sets of Zadeh (1963, 1987, 1978). In other words, the object is the most representative (in the fuzzy sense) of a set of objects, say a fruit, and which we, still using the language of set theory, describe/define by creating a degree of membership, and which is related to the concept of instantiation of OOPLs, or tokens Kosko (1993), Klir and Yuan (1995), (for multivalued logics see Rescher (1969), or Hubey (1997)). The rise of object oriented programming indicates that there has been a slow swinging of the pendulum to the earlier sciences and mathematics. After an initial n years of floundering, computer science has discovered the methods and objects of physics and engineering.

The variables which describe the object can be grouped into *extensive* and *intensive* variables. An extensive variable, say X, is one for which we have $X_s=X_1+X_2$ (where X_s refers to the system and the integer subscripts to the subsystems that comprise it) and intensive variables are those which do not change so that $x_s=x_1=x_2$. For example, if we divide a volume of gas V, into two containers the two containers would contain half each of the original volume and so too with mass. However, the pressure and temperature of the gas would remain the same. In linguistics, lexicon is an extensive variable, but syntax, grammar and typology are intensive. Similarly the educational level of a society is intensive. What is usually meant by "quality" is intensive, and often "quantity" means extensive. However, we can attach a quantitative (i.e. numeric) result to a "quality". For example, it would be quite easy to create a weighted scale for measuring the quality of say personal computers. This points us in the direction of another important concept that has been found to be extremely useful in the development of sciences and intimately a part of Space/ Structure/ Dimension, that of *distance*. However distance is a more general idea which requires a metric space, and which exists in plenty in the physical sciences including in much more abstract form. As is well known we need three dimensions (in a different sense than typical) for mechanical phenomena of physics; time T, space L, and force F (or mass M). For electromagnetic phenomena we add charge Q, and for thermal we need temperature Θ, Ipsen (1960), Isaacson (1975), Pankhurst (1964), White (1979), Gukhman, (1965). This is certainly a generalization of the physical dimension idea, and we need to construct appropriate instruments to measure the values of these dimensions in essentially arbitrary units (such as meter or degrees Fahrenheit). However sometimes we need to select whole sets of them in order to simplify things, such as the SI. Generalizing the concept of measurement, we can then see that the scales can be *ordinal*, *interval*, *ratio* or *absolute* which have now made their way into software engineering from social sciences which had to wrestle with the nonexistence of dimensions such as L, F, T, etc. It's of interest that dimensions are multiplicative and not additive. In any equation, each term must have the same dimensions and these dimensions are multiplicative aggregations of the primitive dimensions. Every equation of physics must be dimensionally homogeneous. What is more amazing is that even the form of the equations about unknown phenomena can be obtained via something as simple as *dimensional analysis*, which seems to untangle the convoluted nonlinearity of real life into chunks of pseudo-independent coordinates or *degrees of freedom* (DOF). Fluid dynamics which is still not solved completely presents very convenient analogical models for socio-economic phenomena, so much so that similitudes from these fields are already creeping into our lexicon e.g. Hofstadter (1995), Rosenau (1990), or even earlier, 'fluid intelligence' (Catell, 1971). Once again, the dynamic stochastic processes via Kolmogorov-Fokker-Planck methods also yield equations for probability current flows which resemble fluid dynamics equations (Jazwinski, 1970; Soong, 1973; Van Kampen, 1976; Hubey, 1993, 1997). It's reasonable to expect that something as complex as evolution should also have mathematical models of this type; Roughgarden (1979), Kojima (1970), Hubey (1996). Ideally we'd like to have metric spaces for every phenomena but so far we have problems constructing any dimensional spaces for nonphysical fields. In the physical sciences by naming things such as velocity (L/T), or Energy (FL or $M(L/T)^2$, we are already grouping seemingly unrelated concepts and unifying knowledge.

We can only be amazed at how it was possible for early linguists who did work on phonology and phonemics to be able to group various phones into such clusters as liquids, nasals, plosives, vowels etc. on the basis of what would normally be called "quality" and which now can be shown to roughly mimic the groupings on the basis of measurements with instruments, and thus on the basis of "quantity". [Please see Figure IV]. The relationship of *quality of sound* (i.e. phones/phonemes) to actual physical parameters like stricture sizes, rates of opening, voicing etc. can be explicitly seen in the figure. Seemingly miraculously, the perception space also looks like the physical articulation/production space, so that a single unified phase space suffices and this phase space was created using dimensional analysis (Hubey, 1994). That this should be so is, in reality, quite expected; if there were no deterministic relationships between the geometry and dynamics of the speech articulators, the quality of the sound, and its perception, we would not be able to use speech for communication. What's interesting is that we can produce such relationships via something as simple as dimensional analysis.

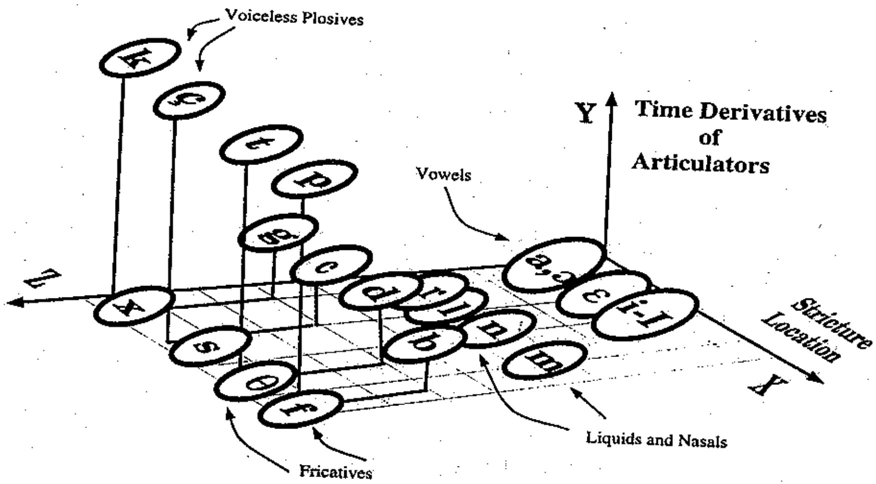


Figure VI: Reproduced from Hubey (1994), Mathematical and Computational Linguistics

Perhaps this is the key that we need to unlock parts of the big puzzle of creating a *dimensional space for perception*. Possession of such a space would give us as powerful a tool as in the study of fluid dynamics by grouping the relevant parameters in such ways as to create dimensionless numbers which we can try to correlate with one another instead of the now pedestrian approach of mindlessly regressing everything against everything else (Dawes, 1994). This idea would be another instantiation of the correspondence principle, that is, the same dimensions of the phenomena in the real world would produce the basis for its perception in the *mental world*. Is it possible that there exist such fundamental dimensions for society, economics, history, AI, and the like? Is it possible to list a number of categories which are fundamental from which many of the other ideas can be derived for such fields as psychology, cognitive science, or sociology? Lacking such things, is it possible that some simple ideas from the sciences that gave us so much can be used as guiding principles? Perhaps one day, we can create a similar set of dimensions for things such as beauty, honesty, discipline, creativity, aesthetics, etc. It doesn't seem as if these would be difficult since they do refer to things in the real world, and some of the more abstract/virtual objects of computer science have already yielded to mathematico-scientific analysis, so that the same concepts now in use in the study of complexity can be used here. For example, we already see that concepts such as *similarity*, *analogy* and *metaphor* are also distances; two similar objects are close to one another in some appropriate perceptual space. The relationships of words such as distance, analogy, metaphor, isomorphism, similarity, and resemblance to each other are explicable on some fuzzy normalized scale, say [0,1] in which the word 'different' would rank as non-zero, whereas metaphor or analogy would be near zero, and same/identical would have to be zero. Thus similarity is a distance in some perceptual space of N dimensions (many of which we cannot name in the social sciences including psychology, and the so called mental phenomena.) Analogies are made in some specially selected single dimension between two vastly different actual or virtual (abstract) objects. So an analogy is a colloquialism for isomorphism (or an "isomorphism is a fully clarified analogy", Polya). Isomorphism itself is a small (or no) distance of a special kind in that it is distance between two structures (i.e. groups). The word 'different' itself means 'large distance' in some perceptual space, and even the words *virtual* or *model*, *prototype*, *artificial*, and *simulation* are expressions of perceptions of small distances, or closeness between two complex real and/or virtual objects. We always think of mathematical models as a kind of an isomorphism between a real structure/object and a mathematical structure/object. In this sense 'folk psychology' itself is a theory of the universe (Churchland, 1994), Davies & Stone, 1995). The problem of 'representation', (for example, Cummins, 1995; Churchland, 1994; Lycan, 1990) a word with rich physical/neurological possibilities, also lends itself to the psycho-linguistic description of this modeling process by which we try to comprehend and communicate our subjective view of reality, although at times it does seem to give birth to the ghost in the machine. As such science itself is mathematical modeling and is naturally about testing the behavior and structure of the model against observed reality. In more complex objects of the real world we observe that they are not simply vectors of independent properties but some elements of the vector are dependent on others and some are hierarchically arranged so that an object is a structure which might be expressible as a special kind of a vector.

In two dimensions of precision/accuracy vs complexity, similarity ranks high in complexity of phenomena but low in precision/accuracy; isomorphism (of the mathematical kind) is high in both complexity and precision/accuracy, whereas the simple concept of distance in metric spaces could be high or low in complexity. Intuitively, dimensional analysis says that we cannot mix terms in which one term is, say, acoustic, and the other electromagnetic. This is the analogue of the philosopher's "category error" which is prominent in the mind-body duality duel (Ryle, 1962). But we already have gained something via dimensional analysis which is already having an effect on disciplines which had no such fundamental dimensions. We now have the dimensions of entropy (order) in terms of the fundamental dimensions of the physical world, and it is our window into the world of complexity. It seems that the complexity of the social sciences is so great that in addition to having to live in and with the usual dimensions of physics (and the physical world) we need something like entropy as a gateway into the world of mental phenomena. The mathematical basis of thermodynamics is much more rigorous now and can be used as an example of a system-theoretic analysis which takes into account previously unaccountable things such as order (entropy), the directionality of real world phenomena etc. and can be found in Owen (1984), Brooks & Wiley (1986), and Von Bertalanffy (1968).

IIIii. Scale (Size, Magnitude, Levels, Layers, Modules):

Scales do matter. It is one of the ways in which we decouple or compartmentalize phenomena, which is really another way of saying 'analysis' since we slice off a piece of a large chunk, and try to study that in detail; when it's time to put the pieces together we have 'synthesis'. These large pieces are part of yet a larger picture so analysis and synthesis are relative concepts; in other words, they are dependent on scale. What might not be so obvious is that this process is related to iteration, repetition, and recursion since, we can also scale things hierarchically according to some criteria (i.e. a change in some variable's values), for example, perturbation expansions for approximate solutions of nonlinear equations often used in physics and engineering. Scaling is used to order things according to size, and can be seen in physics most clearly since the fundamental breakdowns occur according to scales in time and space, and so it is with the social sciences, for see figures VII and VIII].

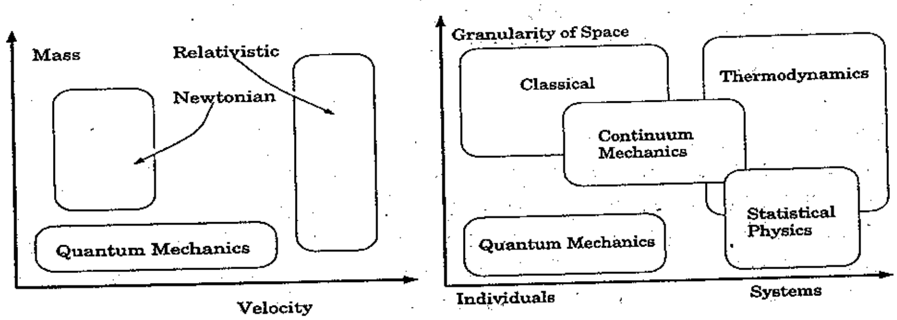


Figure VII: Scaling Phenomena in Physics (Hubey, 1994)

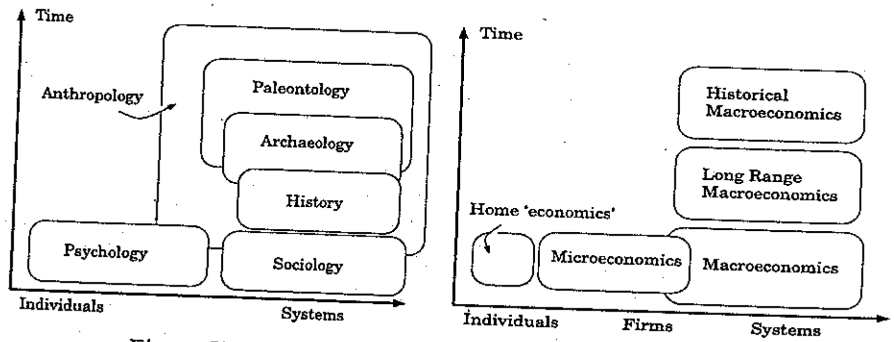
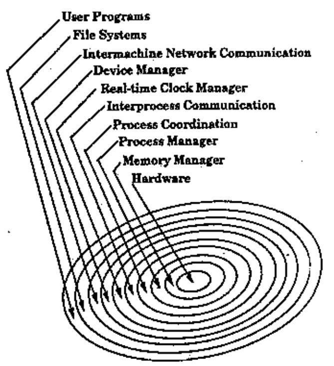
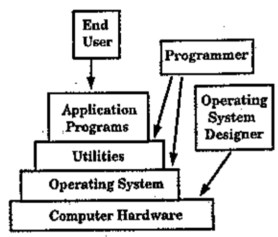


Figure VIII: Scaling in the Social Sciences (Hubey, 1994)

We see the idea of reusability, iteration, repetition, and bootstrapping even in science. In fact, many of the chicken-egg type arguments in philosophy are of the bootstrapped type which ignore layers and levels. We can describe the middle parts of any algorithm usually easily; it's the beginning (initialization) that causes special efforts to be expended. The start up of a digital computer (bootstrapping) is one such event. The very act of doing science is also bootstrapped; we start by observing phenomena at our own layers/levels, in other words, that which is available/accessible to us via our naked senses. Once a theory has been formed that 'explains' this, we can poke and peer further and deeper using instruments that were developed on the basis of earlier knowledge/science. It's this process which gives rise to arguments about whether the theory decides what can be measured or whether we simply make measurements and then theorize later. Both views are caricatures of reality. One has an analogy with computer science. Machine language (ML) had to be used to create the first primitive Assembler, which could then be used to write compilers for High Level Languages (HLLs), and they could be used to write other compilers for other HLLs or even other assemblers. Similarly, the needs of compiler writers and operating system designers spur changes in hardware design of CPUs. Tarski inspired solutions of paradoxes of logic which are solved via languages at different layers is yet another example.

One of the intents of this paper is to present a discussion of fundamental ideas not necessarily based on specific fields or views such as falsificationism(logic), verificationism (probability theory), or language (formal language theory) but rather at a level high enough to examine all possible tools we have used over years to solve problems, especially those (analog/global) ones that almost never seem to make their way into philosophical discussions. Various views of layering and scaling in the computer sciences, social science, mathematics and philosophy can be seen in Figures VI-XI. Exactly the same kind of layering can be seen in the ISO layered protocols for OSI (Figure XII). As another example, there are several layers of structure in geometry; topological, differential, algebraic, affine, etc. Similarly we can see (Zeeman, 1977) several levels of analysis in applied mathematics and dynamics: (i) Singularities, (ii) Fast dynamic (homeostasis), (iii) Slow dynamic (development), (iv) Feedback, (v) Noise, and (vi) Diffusion. Catastrophe theory classifications belong to level (i). Levels (ii), (iii), and (iv) refer to ordinary differential equations, and level (vi) to partial differential equations.





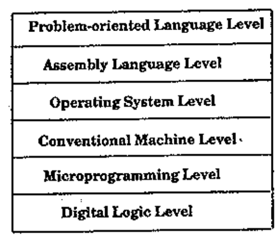


Figure IX: Comer (1989) Figure X Stallings (1993) Figure XI: Tannenbaum (1984)

One might try another (dichotomous) categorization of the various DE methods available for modelling dynamic/kinetic phenomena: (a) Linear vs Nonlinear; (b) Ordinary vs Partial; (c) Homogeneous vs Forced; (d) Deterministic vs Stochastic; (e) Real vs Complex. We note that our original dichotomy could have been differential vs. integral equations, and what is above is only the subset of the differential equations. We could have also easily added the distinction between constant coefficients, time-varying coefficients, random coefficients, etc. In any case, we can show the structure of mathematics as a layered/ modular structure in a grossly aggregated way (Figure XII). In layered programming one can always call on the functions in the lower layers where modules are smaller constituent parts of the layers. As can be seen in the layered model of mathematics (Figure XII), it's via the use of the symbols and structures of the lower layers that bigger modules (which are parts of the upper layers) can be produced. Therefore modules (such as the compartmentalized education of mathematical topics and also physics, engineering and computer science) are parts that comprise the layers. In a sense, then one can draw a dependency tree (such as a prerequisite tree) for the various branches. It is of interest that particular areas of study, say literature or the intuitive psychology studies such as Freud, Jung, etc. may resemble forest, bushes or even grass (single layer).

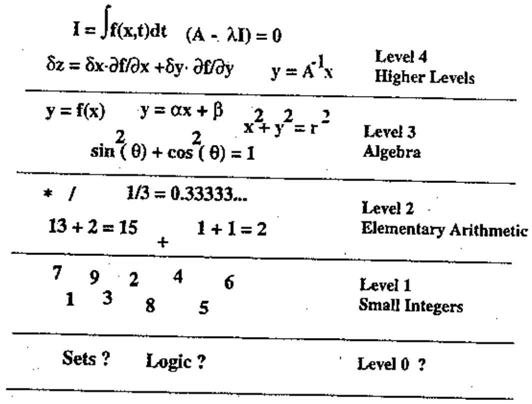


Figure XII: Mathematics Layering

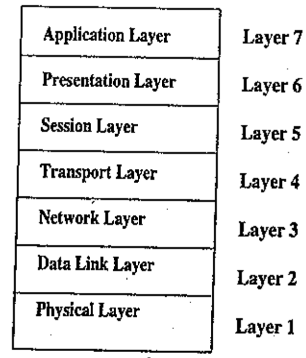
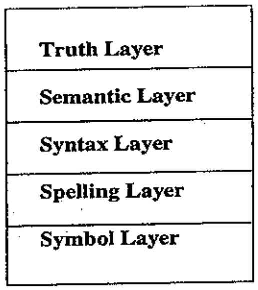


Figure XIII: Computer Networks: The OSI Model, Stallings (1994)

We can show similar scales and levels for the study of something we all love to talk about, natural language. Even here we can see that there are different ways in which we can create these layers. In one of the figures we see a layering according to the standard treatment of languages, and in the other a borrowing of some ideas from formal language theory and computation. Suppose we have a computation engine of sorts, broken up into several layers; for simplicity, assume that the communication is in writing. We might break up the hierarchical layers as shown in Figure XIV. Suppose that the system operates on a received message starting with the lowest layer. First there must exist an abstract synchronization for communication at the lowest layer in that there must be an agreed upon protocol for the symbols to be used for communication. Suppose that the received message is: {<Eετ><λιττελ><βεεγ><γηοτι> <γηοτι>}. Assuming that the English language is being used these symbols will not pass the symbol layer processor since it is looking for the standard English orthography. If we change it so that the message now reads: {<*Eet*> <*littel*> <*beeg*> <*ghoti*> <*ghoti*>}, then it will pass the symbol layer but fail at the spelling layer. If we change it to, {<*Eat*> <*little*> <*big*> <*fish*> <*fish*>}, the syntax layer will halt the message. Suppose sentences like {<*Adjective*><*Noun*><*Verb*><*Adjective*><*Noun*>} pass the syntax layer. Then the sentence {<*Big*><*fish*><*eat*><*little*><*fish*>} will pass the syntax layer but then so will sentences such as {<*Smart*> <*rocks*> <*read*> <*angry*> <*bananas*>}. It's at this point that we get stuck trying to find a machine that can accomplish this; except for a *competent* human being. It is at this layer, that some of the toy expert systems show some proficiency i.e. SHRDLU. There does seem to be a paradox, that although we can write expert



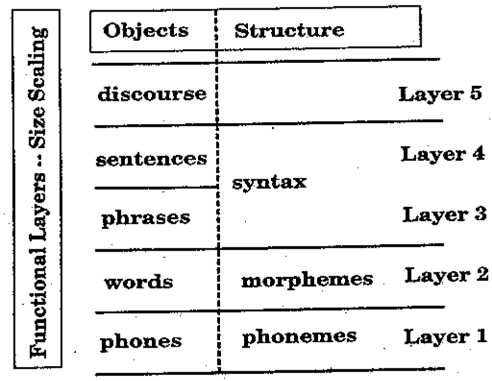


Figure XIV: Language Processing Layers

Figure XV: Layering of Language

systems that can act as geologists, physicians, computer system experts or even artificial mathematicians, we do not seem to be able to give computers *common sense*. Of course, there is no paradox at all; a human is an expert in a huge number of domains. To create an artificial average human brain, maybe even a moronic one, we'd first have to write a huge number of programs for making computers expert at simple things, then put them all together in some systematic and seamless way to create common sense. And common sense is the necessary ingredient for acquiring expertise in a specific domain for human beings. At this point, the line between common knowledge and specialized knowledge become blurry. For example, the sentence <*IBM makes Amigas*> is a meaningful statement for anyone who knows that IBM manufactures computers and that Amiga is a computer, hence it would pass the semantic layer. However, IBM does not manufacture Amigas. This kind of knowledge belongs to an expert domain, at the highest layer, the one I've named the truth layer. If we use this simple natural language model as the model of learning knowledge we can notice that it is at the top two levels that formal logic operates, and Tarski's solution of truth is between these two layers. As for a general model of (scientific) knowledge, we have more problems. Since everything in the universe somehow has to be related to everything else, we have to decouple and compartmentalize knowledge according to some criteria and this criteria can't be other than the strength of the coupling at various scales as we move up and down the multilayered hierarchical tree of knowledge. The strange thing is that once again we have the dependence of the scales on each other. In effect, the way the objects of our study are related to one another is one (if not the only) way in which we can scale, modularize or decouple the phenomena so that we can describe it. Fractals are a peculiarly beautiful and simple example of the case in which there is similarity at different scales. This self-similar art can be seen in 16th century Ottoman works.

Imprecisely modeling upon the analogies of loosely-coupled computer networks there are theories of the mind/brain, which are also similar to the agents of economics theory, and social interactions of such agents. Among the earliest such examples we have Minsky (1985) from the point of view of AI, Ornstein (1986) from psychology, and Gazzaniga (1985) from neurophysics. Of course, such views are now common and have been elaborated in many ways and by many people, for example, Dennett (1991), Churchland (1995), unifying the mind with brain; Eysenck (1967), personality based on biology; Gardner (1983), many different types of intelligence; Powers (1989), layers of mind/brain based on analog control paradigm (i.e. cybernetics) instead of the digital computer model; Churchland and Sejnowski (1993), the modern computational brain/mind; Eccles (1988), evolution of self via the brain; Hobson (1988), modern (unfreudian) views of dreams, and Newell (1990), levels/hierarchies/ bands/scales of events for cognition. Of course, there is the likelihood that the brain will turn out to be more complex than the structures which we now use to describe it, as perhaps can be evidenced from our use of language. For example, patterns of some sentences turn out to be superpositions of more than one single structure such as a stack, superimposed on a barrel shifter which is represented as levels Hubey (1994). The concept of life phenomena obeying laws of scales in addition to being intimately related to dimensional analysis (gravitational forces vs strengths of materials) can be seen in Schmidt-Nielsen (1984) as a simple application in biology. Further research into the concept of dimension, scale and measurement in the social sciences (and now software engineering) can be seen and is discussed in Bock (1968), Gorden (1977), Lodge (1981), Manning (1968), Maranell (1974), Schiffman (1981), as well as in the books by the originators/-founders Torgerson (1958), and Stevens (1951). As can be seen, we often do not, and cannot find absolute/ratio scale measurements, and have to be resigned to relative(interval) measurements or sometimes even to rank scales (simple comparisons). The Turing test is one of simple comparison (please see appendix). Then making models, and testing them becomes even more problematic. The discussion and measurement of intelligence falls in this category of measurement scales, and the pitfalls and errors of standard arguments for/against various views can be found in Hubey (1996). Levels as specifically applied to problems of cognition, intelligence, and control, and as unifiers of complex phenomena involving intelligent systems (beings), and analogies to computer levels can be found in Newell (1990). Specifically, Newell bases the levels on hierarchies, which is based on Simon's ideas on stability of hierarchical systems. Newell, also has explicit time scales of human actions; the Biological (neural), Cognitive, Rational, and Social bands. What is needed is the explicit recognition that not only should we use equations appropriate for the level at which phenomena occur, but that some phenomena, at some levels, influence phenomena at other levels at various magnitudes also dependent on scale, and that we might need to (not necessarily additively) superimpose several classical/standard structures, which is really an admission of the complexity of the phenomena being studied and its hierarchical nature. A simple taxonomic structure of collections of computers based on the distances between the processors, ranging from data flow machines to wide-area networks can be seen in Tanenbaum (1988). A similar layered taxonomy for the mind based on distance starting with Angstrom units up to a meter or so, in addition to the appropriate scientific tools for studying each level, can be seen in Churchland (Posner, 1993). For handling such complexity at various levels more appropriate tools have been developed, for example, *Hierarchical Information Theory* of Cumming (see Brooks & Wiley, 1986). Not unnaturally, these ideas are related to probability theory, information theory, and various types of entropy, in terms of which many concepts on complexity have been developed (see for example, Li & Vitanyi, 1993; Chaitin, 1987). Our mathematics, as fruitful, and as marvelous as it is, is still not up to the task in many ways. For example, in deriving the basic laws of continuum mechanics level description of physics, we use derivatives, and as we well know, the definition of derivative has some small quantities approaching zero but these derivatives in continuum mechanics do not 'see' the quantum nature of materials. In effect, continuum mechanics is decoupled from and independent of quantum mechanics. Similarly, we can derive deterministic laws by averaging over stochastic equations, but to make the results more realistic we have to add more noise after averaging out the small fluctuations to derive the deterministic laws of physics. We do not yet have the means to average which only gets rid of small scale fluctuations and still leaves stochastic equations which still have large scale fluctuations in them. We can see that society affects every individual over a long period of time, and the change in the individuals' character/personality when passed onto the next generation, eventually leads to change in the character of society, which again continues to influence every individual. At longer time scales, these account for long range oscillations of history, and at even longer ranges for biological evolution (Hubey, 1996a). When generalizing from micro-laws (such as in differential equation models) to macro-laws, averaging the solution is not equivalent to solving for the averaged equations, and thus generalizing from one level to the next level is a complicated effort, the laws of which are not completely understood (Van Kampen, 1976; Hubey, 1993).

And, of course, distance/space and scale concepts can co-occur to increase the complexity of the phenomena. It's in the scales much greater and much smaller than the level which our senses allow that we might have some justification for asserting the truth (in some sense) of the statement that *reality* and *appearance* are not the same, without being mystical about it. In this sense there is some germ of truth for such mystical philosophical movements, which in any case are no longer appropriate since we can simply call high school science for help in clarifying expositions of the relevant concepts. The differences in appearance vs reality are much more pronounced in psychology since it's here that we can plainly see that personality (a particular facade or a modus operandi) and character (inner characterization of a person) are not the same. Indeed it's when the disparity (distance again) is large that we can see that the person is either a confidence artist or a mentally ill individual. Cognitive dissonance (Posner, 1993) is obviously an implicit call to the concept of distance.

III.iii. Pattern (Relation,Function,Interaction,Dependence,Propensity, Coherence,Correlation):

It's natural for us to look for pattern, coherence, correlation, structure, functional dependence, etc. among objects in the real world in mathematical modeling. And it's the study of this search for pattern or structure that comprises the Holy Grail of the latter part of this century, and our toolkit for finding, and describing such patterns has been enriched by the mathematics of the nonlinear, chaos, fractals, and complexity. The simplest and the commonest forms of pattern, in the unreal idealistic world of classical physics, are the deterministic ones given by functions such as $y=f(x)$. We can see that the concept of repetition, reusability, iteration is built into it, since we can always pick any number for x, and compute the value of y. In one sense it resembles a function or procedure call of programming, since they too are reusable and deterministic. In classical physics (and hence engineering/technology) most of the relationships, with the exception of fluid dynamics, are of this type. We have separate (sometimes uncoupled) sets of equations for behavior at different scales in physics and engineering. For example, we have approximations for *lumped-parameter systems* derived from equations at the continuum level, and while we're aware of the quantized nature of the materials we go on using the continuum level descriptions which are completely appropriate for the scale of our normal lives and for which our naked senses are appropriate. There are some very basic and fundamental concepts that are used throughout the sciences and various branches of mathematics in search for pattern. They are given in Appendix 0.

IVa. Where Is the Complexity? : Old Problems, New Paradigms

We're now in a position to ask what complexity is and where we can find it in a more serious tone. Complexity has always been with us everywhere and we have been discussing it all along. Even the three dimensions just outlined, if we attempt to produce a space from them, must be drawn on the basis of some kind of an intensity variable, and for all three dimensions the best candidate is some kind of complexity. Spaces have complexities, functions/relations have complexities, and going from one scale to another or even better, reducing description at one scale to another scale is dependent upon the complexity of the phenomena. If we attempt to produce high level descriptions of extremely complex objects, we will be forced to trade-off accuracy. From another (more constrained) perspective 'complexity' is simply a new set of tools (paradigms) which have been made possible by the advances in mathematics, science and technology. They are primarily mathematical tools for dealing with nonlinearity, and which are needed to study the nonlinearity/complexity in the real world. As such we can give them names, such as; *Chaos Theory*, Strogatz (1994), Schroder (1991), Ott (1993), Prigogine & Stengers (1984), Peitgen (1992), Schroeder (1991); *Catastrophe Theory*, Thom (1975), Zeeman (1977), Gilmore (1981), Woodcock & Davis (1978); *Fuzzy or Many-Valued Logic*, Zadeh (1963, 1978, 1987), Kosko (1993), Klir & Folger (1988), Klir & Yuan (1995), Rescher (1969); *Cellular Automata* and *Self-Reproducing Automata*, Toffolli & Margolus (1985), ideas of Von Neuman, Burks (1966), related topics such as Forrest (1991), and Devlin (1991), and finally *Complexity Theory* itself, Papadimetrou (1994), Chaitin (1987), Li & Vitanyi (1993), Zurek (1995). Intuitively, we can relate complexity to difficulty. Something that is formalizable, then, becomes noncomplex, for as conventional wisdom in mathematics/logic has it, any formal system is ripe for automation, and anything that can be done by a computer is no longer interesting, and according to some, does not require 'intelligence'. Of course, we could be ripe for a change in the meaning of 'intelligence'. Just as we need more ability, capability, or power to be able to solve more difficult problems, we can relate complexity to the inherent difficulty (usually of nonlinear) problems, and to power of the new tools with which we attempt to solve such problems. There are several types of difficulty; one kind is simply taking much time to solve although there is a solution method. Yet another kind is the type that comes from novelty and for which we have yet no solutions. For example, walking is not difficult for most people (except for those with physical disabilities), however walking from say Paris to Vladivostok would be considered difficult (and probably foolhardy) by most people. Trying to fly was of the latter kind of a difficulty, that is, until the Wright brothers. In software engineering, programming tasks can be broken down into run-of-the-mill computational problems vs new one-time projects (of which the most difficult might be real-time embedded systems). It is in this sense that complexity measures in software science are not all measuring the same thing. Computational (or time) complexity is a measure of the time necessary to compute an algorithm (and hence solve a problem of some size S) and is related to the first kind of difficulty, whereas algorithmic complexity is a measure of the length of the algorithm (if such an algorithm exists) that produces an output so that it is a measure of the second kind of difficulty. It's essentially our intuitive concept of a problem not having a solution. Since we want a given problem to always have a given level of difficulty, this definition is not satisfactory since it would mean that this kind of complexity is a function of our abilities instead of being an inherent property of the problem. In this sense it might be an attribute, in the sense that we attribute this to the problem, and changes in our abilities can result in reattribution. In any case, we have another example of a relative measure.

Complexity, then, in an intuitive sense is a product of the dimensionality of the phenomena. A simple way to measure complexity is simply the number of (non-regular) dimensions of the phenomena by which we are excluding simple increases in the dimensions of a phenomena, say in linear algebra from five to seven, since the only increase here is in size, the first kind of difficulty. The increase must be multiplicative (as in Ockham's Razor) in some way, as is implied for example, in simple Cartesian spaces (products) of sets of the form $A_1\ A_2\ \ldots A_n$. Thus we can (also intuitively) see how dimensional analysis reduces the complexity of a problem. It does this by *multiplicatively* and *nonlinearly* reducing the dimensionality of the problem. For example, for a complex empirical problem in fluid dynamics it is much easier to work with three dimensions, say Weber number $We=\rho V^2L/\gamma$, Strouhal number, $St=\omega L/V$, and Prandtl number $Pr=cp/\kappa$, than with 8 variables, ρ=density, V= velocity, L=Length, =viscosity,ω=angular velocity,γ=surface tension, cp=specific heat,and κ= thermal conductivity [White,1979]. The strange thing to note is that what we saw earlier in the spaces for philosophy and epistemology also happens here. There is no reason to expect each variable to show up only once in the dimensionless groups. It is by the very fact that these dimensionless numbers (products of basic/fundamental variables) show up in various forms that simplifies the empirical relationships to look for, and it is the fact that the variables can show up in various dimensionless numbers that tells us that there's nonlinearity. It causes a dimensional collapse/simplification at the expense of increasing the complexity of the dimensions themselves (yet another trade-off). However in empirical tests, we are more than happy to have this trade-off. Similar simplifications would be (or should be welcomed) in the social sciences, and philosophy. The real problem, of course, is to find the dimensions of philosophy or even epistemology.

Another simple way of going about defining complexity is to count the number of independent variables we need to describe the phenomena. Here we can see the interplay of dimension and precision. For example, for point masses of classical conservative systems (mechanics) we need only displacements and momenta for each particle (Lagrangian or Hamiltonian description). Therefore, for a complete description of an ensemble of N particles we need 2N dimensions. However for lumped-parameter systems (real objects) we need 6 DOFs, three for its position in space (translation), and three for the rotations possible in space (pitch, roll, yaw). However if we try the Eulerian description we replace the coordinate systems attached to each particle with a coordinate system attached to the universe and thus produce continuum mechanics (elasticity theory, fluid dynamics, waves and fields) in which we describe (mathematically, of course) some relevant properties over 3D space. The number of dimensions is greatly reduced but so is the resolving power of the system. As a matter of fact, we cannot fully solve any but the simplest problems, i.e. those having some kind of reducibility such as due to isotropy, homogeneity, linearity, and above all, to some regular simplified geometries and shapes. Of course, we can see from the previous discussion that complexity in the real world really can't be anything other than a product measure of the dimensionality of the problem. It's the size/scale of the problem times the inherent difficulty of the problem, which might be written as SSP (*Space* x *Scale* x *Pattern*). Of course, this has long been recognized in the real i.e. *Time* x *Place* x *Setting* (actually *Zaman* x *Mekan* x *Imkan*) for these are the things that determine whether a child becomes a scientist or a thief. And finally, we must be able to represent methods, structures, solutions in this space according to magnitude and this magnitude should be the intensity of something, and here again recursively, the best candidate is some measure of complexity itself for each of the three dimensions, *Space, Scale,* and *Pattern*. It would be extremely difficult to construct the 3D space from the dimensions just discussed since it would need a distance metric to enable us to put disparate ideas together and somehow manage to measure the intensity or the power of various mathematical (and possibly other) methods. However, things simplify (as expected) if we increase the dimensionality of the space.

However, we can construct another kind of space based on dichotomous taxonomy, which is not be restricted by the dimensionality of the space, and yet possesses an intuitive strength since it can be embedded in 3D. Returning to the original taxonomy tree, it is literally the case, that as we add more criteria for taxonomic structure, we create more and more dimensions, and that the serial walks figuratively enter into higher and higher dimensions of the space of thought (see Figure XVI). What we are seeing is an unfolding of the phenomenology of thought as we peer deeper and deeper into space representing the various categories especially if we drop the simplified binary view and treat the dimensions as normalized intensities of various views. In this sense, the sequential nodes in this path (space) are each separated by a distance of one unit, since only one of the dimensions of this binary/taxonomic space of thought changes at a time. But as we proceed to add more and more criteria to the subject matter being studied, we hit upon similar ideas again and again, almost like a regular periodic visitation to a region of space as in ergodic processes. In the objects example (Figs. I-III), we have not weighted any of the edges or nodes (because we have not given an weight to the importance of any of the criteria). However if we wanted to produce such spaces for discussing various philosophical schools we would assign weights to the edges. Indeed this is implicitly done since some ideas are always thought of as the basis for the whole school of thought. Often, one starts with some fundamental assumption and extrapolates relentlessly on the basis of this putative criterion the obviousness of which is allegedly clear to every reader.

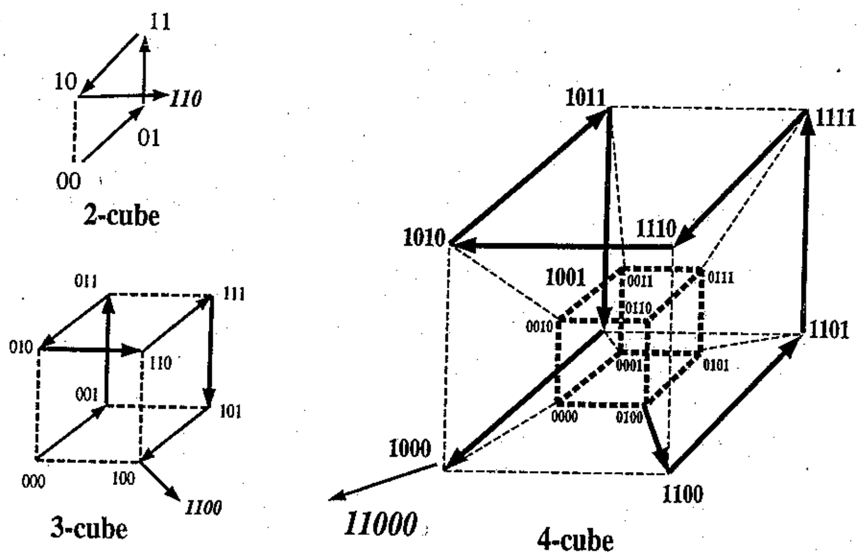


Figure XVI: Unfolding of Higher Dimensions of Taxonomy via Hubey Traversals

We can redraw the taxonomy tree and make connections to show more than what is usually shown as in Figure XVII. What we see at each level is hypercubes of higher and higher dimensions (although they are hardly cubes of any kind in the lower dimensions). At the fourth level we'd need to draw a 4D hypercube consisting of two 3D cubes (Fig. XVI). So the tree is not a very natural way to visualize such dichotomous taxonomies. Aside from the possibility of using the traversal through higher and higher dimensions for such taxonomies (Fig. XVI) or making trees (Fig. XVII) we can use these ideas to produce a topological space for philosophy. The space happens to be a torus, and is based on the idea of the Gray code, with the Hamming distance being the metric for distance in this space. The only thing that has changed is that we can now show a topological surface, and this, of course, is at the highest level, and is therefore lacking in precision. However it is general, a true metric space and a topology that is intuitively comprehensible and graspable for any number of taxonomic

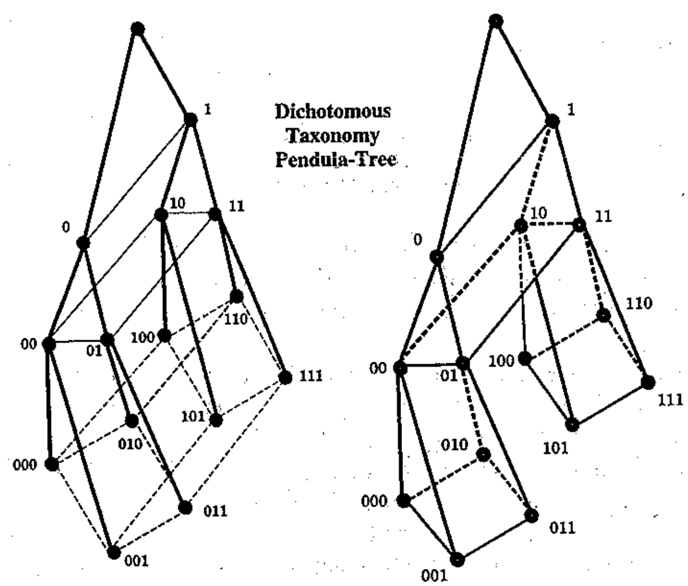
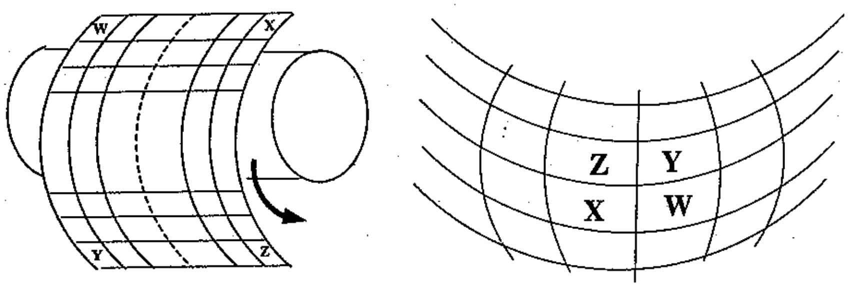


Figure XVII: Simplified Tree of the Taxonomic Hypercube
(In the figure on the left we can see the higher dimensional hypercubes at each level. In the diagram on the right, we only see the wedge repeated at each level. Connections of the wedges produces the hyper cubes of higher dimensions)

dimensions, since they can all be mapped to the surface of a torus. We simply use the binary bitstrings (vectors) to construct a Karnaugh map and then simply wrap the map onto a torus as shown (Fig. XVII) so that it is plain that the edges of the Karnaugh map are neighbors of the opposite edges Hubey (1994).The surface of the torus is natural space for classification, study or analysis of any kind of a system including, and especially the systems of philosophical thought in which their characteristics can be described by yes/no answers to a set of questions.



Topology of Dichotomous Taxonomy:
Metric Space for
Philosophical Thought

Figure XVIII: *The Karnaugh map is wrapped onto a cylinder (left), and then the two ends of the cylinder are joined to produce a torus (right) so that the corners of the map are now neighbors (i.e. one unit distance using the Hamming metric)*

It could seem strange that whatever complexity seems to be, evidently it can be seen as many things, and that we are no closer to defining or understanding it. However, this very fact, this capability of the same (abstract) object to be seen in many different ways, like the way the elephant was observed by the four blind men, might be yet be another measure of the complexity of the phenomenon. It could very easily be due to complexity that we are forced to view light as both a particle and a wave, a wavicle. It is not for no reason that physics exists at different levels of description. It should not be so difficult to accept that something as complicated as a society comprised of the interactions of intelligent beings can be seen in and described many different ways. Like the modern idea of databases (and modern operating systems) in which we can have several logical (conceptual) views of the same object, these are all views of the same reality. We can hope that one day we can derive any of these views from any other via mathematical operations.

Natural language works reasonably well because its expressive power is high. However its precision is weak and it is for this reason that we like to do science in a language whose precision is better than natural language. It is reasonable to expect that the definitions of primitives must be encoded in the axioms of systems in the same way that meaning in natural languages can be gleaned from the structure that is embedded in this serial stream of words that we call language. (Actually parallel processing is going on during the recognition of words, and creating meanings from strings of words.) In that sense it might be possible to define the terms or primitives in relation to each other. A simple version of this can be found in Hubey (1997). The method of constructing the structure of *chaotic attractors* from the time series of only a single variable is an inverse operation, in a sense, of serialization, since we recover the higher dimensional structures from a single time series. In that sense, the multi-dimensional views of the binary hypercubes (which represent some categorization of the type which serves as the beginnings of all sciences and for much philosophical discussion) are equivalent to a serialization (a particular weighted path through the dimensions). Since there are many such paths, one might ask if there are preferable ones. For sure, as far as philosophical discussions go, the paths are important, since what seems to happen is that some particular aspect of a phenomena is usually found by some particular philosophical school as the most important discovery, and on the basis of which one goes through a particular walk thorough the edges so that "everything" is explained in terms of this wonderful (and heretofore unknown) discovery. If everything is related to everything else (as depictable on the hypercube) then it's easy to comprehend how everything can be "explained" on the basis of one of them. Therefore this space also gives us the capability of distinguishing philosophies, since what we need to do is assign weights to the edges, and then sum the traversals. Indeed, in assigning particular importance to a specific idea, a philosopher (or philosophy) has already weighted his/her perspective, and we are only recognizing it. In some unfortunate cases such as in schools of psychoanalysis,or caricature versions of behaviorism, or materialism/reductionism, it is obvious that like in numerology or astrology, or like a quack doctor who prescribes castor oil for every ailment, there is a tendency to stick to some simple (and usually single) principle/variable. Naturally, we might be accused of having done so here, since mathematics may be considered by some to be such a simple, and single entity. In that sense, these methods are still in the same class of attempts to portray/communicate/encrypt multidimensional information into a single serial channel, and also to recover the original higher dimensional structure from serial data. It is reasonably plain that logic (and formal mathematics) is a serialized version of a multi-dimensional reality. Why phenomena that is naturally multidimensional should be turned into a serial stream for alleged rigor is most likely the residue of logicism and set theory. There are yet other possibilities for complexity, for example, as in defining intuitive complexity as a product of an extensive variable and an intensive variable, as might be the case for measuring intelligence (Hubey, 1996), or as a product of precision and power since no matter what we precision increases the cost (see also Appendix I).

The overall picture can be clarified by an example delineating how philosophy is different from and also like science. They (like art) are both creative endeavors; they are also holistic (system building), and they both go beyond the information given (observation) by making inductive generalizations. Indeed it's impossible not only to do science, but also to have common sense without inductive generalization. In mathematics and philosophy this problem is usually tucked away with phrases such as *a priori*, *axiom*, *true in a given system*, etc. It's impossible to justify deductive logic without using either (i) inductive logic or (ii) falsification. How they are different is: philosophy uses a relatively small knowledge base, extrapolates far, works at a high level (at low precision with natural language metaphors), and uses almost no other tools than logic (which is still not considered math by some people). Science is based much more on empirical knowledge (of the measured kind), makes small extrapolations (although going beyond the mere information given in positing "theories"), and works at the detailed level (high precision). From an historical perspective, philosophy at one time was a (low level/intensity) science (since there was no science), and now many (precise, high intensity) sciences have branched off philosophy leaving it, some might say, sterile and bereft of any interest. And yet one more perspective would be to say that modern philosophy is science for the thinking but scientifically illiterate masses since it follows science but lags behind it by some time. It is a holistic endeavor which tries to provide meaning by extrapolating from science, technology, psychology, religion, ethics, and tries to provide a complete system, a way of life. In this sense, it will have to change to meet the challenge of the new tools. It is obviously not the case that science is not about doing the same boring thing all the time, or that art is about nothing but novelty. They are similar in that both can be creative and also in that both require lots of learning by rote. In one case, one seeks a single solution (sort of like looking for a needle in a haystack) of all possible solutions; in the other one looks for a solution that is different than all the rest (otherwise it would be a copy and that's obviously a no-no for art). The main difference is that one is reasonably well-understood and has rules by which one plays (which are reasonably strict) and in the other one gains by breaking the rules (smashing the taboos, so to speak). Of course, science might also go through these fashionable phases (a la Kuhn) as art. A statement like "a cloud of initial points, moving through space....." in a description of the mathematics of stochastic flow/

behavior is highly suggestive (Van Kampen, 1976). Is it an analogy, a description, a metaphor, a similitude, a model, poetry or science? Why is it not *creative art*, like poetry? Is art creative, and is creativity not mechanizable? (i.e. an artificial scientist possible?) Is science not creative and hence not art? Is art only useless beauty? Similarly, is philosophy cheap science or is science nuts & bolts philosophy? We have the gradations/scale/levels/layers again. Is science broken down into math and applied math (AM)? Then physics must be applied math. If so is physics applied-applied math (A^2M)? Is engineering then applied-applied-applied math (A^3M) ? We note that the *multiplicative nature* of the acronym A^nM is not totally inappropriate since the A is a modifier (say some number between zero and one). In some ways, philosophy is like art; only originality seems to count, not getting at the facts and reasonable explanations. There doesn't seem to be anything that some philosopher won't defend. Of course, this too is changing, and it is becoming more focused under the relentless advance of science, and especially in the fields of intelligence, consciousness, awareness, perception, etc. In passing we note that if we want to peer at phenomena at basic levels, in addition to Churchland's "observation vocabulary" we can also see an "analogical reservoir" to describe poorly understood things in terms of better or more common things, and this analogical reservoir shifts slowly over time to analogies from physics, which is one specific application (model/isomorphism) of a mathematical structure to a real world phenomena. There seems to be a time lag effect between reality and perception. This lag of perception behind reality is especially felt in the social sciences such as history and in the comparison of philosophical movements after some large scale change has taken place usually influenced by science and technology.

To create distances between art, science, and philosophy, we simply select some criteria on the basis of which the three are alike and unlike. It's not necessary that the criteria be exhaustive although such set of criteria would give much better results. This is similar in spirit to the idea of distinctive features in linguistics. Suppose philosophy and science have N alike features and M unlike features. Then the (similarity) distance between the two is $S(p,s) = N/(M+N)$. Since by distance we mean similarity, then the *difference* is $D(p,s) = M/(N+M)$. One can readily see that $S(p,s)=S(s,p)$ and furthermore that since $S(p,s) + D(p,s)=1$, we have a *normalized local metric space*. It's local because it doesn't take into account similarities of these fields to anything else such as apples, unicorns, or calculus. In that sense, this idea is the stuff out of which common sense is made and which people use to categorize objects. To compare apples and oranges we normally zoom immediately to the relevant parameters, and don't care about the irrelevant ones. This is because we are experts in common sense, in the same way that a chess grandmaster "chunks" whole boards instead of looking at the detail on the board. The inferencing mechanism is also a part of our memory and is learned. This common sense, aside from being a seamless integration of a gigantic number of expert systems, is also scale dependent and it (coded into our neurons) is suited to the scale of observation of our naked senses. It is this hidden reality (from our naked senses) which until quite recently kept giving philosophers and mystics grist for their pronunciations of the (alleged) dichotomy of reality vs appearance. The dichotomy, if it exists, is between observations with the naked senses on our everyday scale, with the observations (and theories emanating from them) on the basis of more formally developed methods. This we call education.

IVb. Nature of Reality, and the Mind's Place in it.

We can go full circle and look again at the problem of knowledge, how it affects society, and the role of the mind in it. Historically, the best technology of the day was always used as a metaphor for the brain/mind. So it was with clockworks, telephone switches, then digital computers, and now finally ANNs [see for example, Johson-Laird,1988]. The brain is far from what we today consider to be the standard SISD (Single Instruction stream, Single Data stream) computer (i.e. the standard Eckert-Mauchly-Neumann computer). The metaphor is still continuing with the *quantum brain*. Of course, quantum mechanics is not the only way to produce non-determinism, uncertainty or unpredictability. We can simply add (or multiply with) noise to represent errors in the neural circuits, or if we want a more esoteric analogy from the new mathematics we can use deterministic-chaos. Indeed chaos (discretized iteration) is about propagation of errors in computation so that it is akin to randomness. Indeed, probability theory never gave exact specifications or algorithms for producing dynamic or static random numbers which met the specifications implicit in the properties of the density functions. The algorithms for producing them were producing pseudo-random numbers. In any case, we have now advanced one more step in the right direction. So we now have both determinism and uncertainty (or unpredictability) at least at the lowest most detailed levels. It is really unpredictability that philosophers who still espouse free will are talking about when they take a non-deterministic stance (i.e. we are not machines). But very complex machines have lots of ways in which they can make errors. Simply put, if the probability of error of a single component of a structure (per unit time) is φ, and if the complete errorless functioning of the whole system depends on all the parts, then if there are N parts then the probability that all the components will be working, and hence the system is working correctly is φ^N which for any realistic value of φ is a very small number. The brain itself must have a tremendous kind of redundancy, and of course, we know that there is much robustness in ANNs which leads us to believe that this technology today is the closest analogy of the brain/mind.

To see the role of the mind/brain in the universe, split the universe into two groups; one is "I" (for each intelligent entity) and the other, the rest of the ensemble. From the rest of the ensemble we'll only get averages (i.e. moments from some unknown probability distribution). This is the basic thermodynamic approximation of statistical physics in which we take a representative particle, and model the effects of the rest of the ensemble on this particle as the 'heat bath'. This metaphor means a break with logic for philosophers, and an acquaintance with probability theory and statistical physics (or thermodynamics). For the linguists it means that the obsession with the linguistic probing as an end to itself must must be replaced with a view of language as a serialization of multidimensional structure. If anything, the discretization of the analog world causes the quantization error. For example, the sorites paradoxes of logic are quantization errors. In the same way that we can construct the structure of the multidimensional attractor in chaotic flows from a time series of a single variable, we construct many dimensional concrete and abstract structures from the serial acoustic communication data that we call natural language. In the same way that we can envision complex structures producing chaotic time series, we have imagine via analogy that the multidimensional world (the representation) that has been constructed in our brain via the connections of the neurons, can and does produce a serial/single dimensional acoustic data which can be decoded by the hearer. In the not too distant past, the whole analysis would have been of a probabilistic and analog nature. To a physicist a toss of a die is a physical problem whose solution is not computable because of the uncertainty in the initial condition, the impulse imparted to the die, and the lack of knowledge of the other relevant parameters such as the coefficient of restitution, the wind speed and direction, etc. In such complex problems we can now use fuzzy logic, which fills in the gap between classical logic and probability theory very nicely. It gives us another way of looking at uncertainty without the baggage of stochastic processes. A related paradigm of complexity and nonlinearity is that of "deterministic chaos", a process which behaves like it's random but is purely deterministic. This is yet another source of uncertainty, i.e. error in computation. This paradigm, of course, that of having determinism and yet not predictability is attractive to philosophies of the mind since until then the only choice we seemed to have were equally undesirable, that is the choice between randomness(unpredictability) vs determinism(predictability via Laplace's demon).

Now we have a metaphor for human free will, which has determinism and yet not predictability. Naturally, we don't want to be 'machines' even if we are. But so it is with the rationality assumption (whatever it means), and with the conflicts between our egos, and ids. There is always noise in the data of the real world. This noise can be modeled via fuzzy logic instead of probability theory, since fuzzy logic is about analogical and approximate reasoning, as well as uncertainty, and error. Similarly another new paradigm for modeling dynamic uncertainty, chaos, can be used. Indeed in some ways it is even better than stochastic analysis. We, humans, have too much pride to allow that our minds (brains) are mere machines (i.e. deterministic mechanisms or computation engines) and yet we cannot allow them to be random either. Deterministic Chaos comes to the rescue, since as its full name suggests, the random/deterministic dichotomy is being broken apart; we no longer have to connect unpredictability (uncertainty, non-machine-like behavior) with randomness or determinism with predictable machines (like computers), so we can have our cake and eat it too. In the same sense, the paradigms of cellular automatons, that is, simple rules producing seemingly great complexity, or even randomness (i.e. the chaos game) producing order are great tools for the mind, since like the engram (modeled after the hologram), we can see order being created out of disorder (as in evolution) or seemingly complex things in nature being created out of simple rules.

So we have a whole system, which consists of the world of objects W, a single observer I, the ensemble of observers E, the perception channels P, a serial communication channel p, and a communication language, λ. Therefore we have the structure $\{W,I,E,p,c,\lambda\}$ where $I \in E$, $E \subset W$, and $p \in P$. The essence of the PMM-PLL divide can be found here. The various philosophies can also be discussed in terms of these. Those fields that have slowly split off from philosophy and have branched off into their own and have made progress (ranging from physics to economics) have, then via a feedback mechanism, influenced the direction of whatever is left of philosophy. The leading edge of the old undifferentiated mass called philosophy, in other words mathematics, has spurted ahead of the others, with physics (engineering/technology) close behind, and the human "sciences" following them. In this rough description of its history we can almost see all the great isms of the past centuries. And we can probably predict the future to some extent. Ethics is already a part of economics and sociology. Logic is already a part of mathematics, and computer science as far as usage goes. The rest of logic serves as an ideal language (like ideal gas theory) for linguistics and formal theory of languages, and allows one to discuss things with more rigor at the language level by clarifying the syntax (but also causing some of its own problems e.g. sorites quantization paradoxes). Epistemology (including tests for intelligence, the manufacture of intelligent machines, learning machines etc.) will become closely allied with computer science, neurophysics, neurology, and branches of psychology. Because of the tremendous amount of money involved in a capitalist, free-market economy, the advertising business is already turning aesthetics, and fashion (both naturally tied to emotions) into a science with about as much validity as psychology since the same statistical tools are being used. That leaves only metaphysics, and that will always be around since philosophy throughout its history has always tended to occupy the no-man's land beyond science, bordering on science fiction. Of course, it doesn't mean that it has to be non-rational. Rather the emphasis is on speculation which is really another word for nonmathematical extrapolation based on analogy, logic, similarity and metaphor. As such until everything is known (which seems as impossible as the end of time) there will always be room for speculation. And philosophy will serve, as it has in the past, as an integrated holistic view of the universe and as such a whole way of viewing the world, the past and the future, and hence a way of living the present. As such it has to be based on what is known (science), and can be considered to be a watered-down or dumbed-down version of science whenever it deals with topics which have the potential of empirical answers. It cannot really be any other way because there is too much to know for anyone. Even around the time of the Renaissance one could know practically everything that was known; now it's impossible. Therefore in the most extreme representation (in the same way as philosophy itself paints the universe), a philosopher is one who knows nothing about everything, whereas an expert is one who knows everything about nothing. And this sentiment can also be described mathematically; the latter is a Dirac delta function (roughly infinite in value only at a point, and zero everywhere else), but summing to a normalized unity. One way is to define the Dirac function is as a limit of a Gaussian with the variance approaching zero. Therefore we can then easily define the generalist/philosopher using the same Gaussian but with the variance approaching infinity, so that the value of the function will be essentially zero everywhere but still summing to unity. And this too seems to be one of the most fundamental trade-offs in the knowledge game. We have fixed and finite mental capacity and that capacity can be allocated to knowing in different ways.

In this grand scheme of things, the new science of complexity just provides new mathematical tools for studying the universe, and provides us with examples, and verbal/informal existence proofs and models of the many ways incomprehensibles may behave.