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Dynamical Equations of a Simple, Closed Socio-Economic System

by

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Introduction

In the usual treatment of the macro-economy it is common to split up the total final output Y(t), into investment goods I(t), and consumption goods C(t). Similarly the input into the economy is split up into the "primary" inputs, the service contribution of the *Nonhuman Potential (Stocks* N(t), which is called capital *flux* (flow) K(t) in this paper and, the contribution of the *Human Potential*, H(t), we call *labor flux* L(t). In addition, sometimes, the natural resources R(t) (raw materials) and the Intermediate Goods V(t), are taken into account. In earlier models, Hubey and Chichester [2,3] modeled the effects of the *primary outputs* (Investment-Consumption) on each other [2] and the Capital- Labor effects were modeled in [3]. In this paper, a complete closed simple-economy is modeled. It will be shown that the raw resources will enter into this model in a very natural way.

This system has no government i.e. no taxes and no governmental spending; no monetary saving i.e. we start off with an economy which already has Nonhuman Potential and whether this Potential is managed by the non-existent workers of the "disappeared state" or is managed by the "invisible hand" has no effect on this model; no monetary effects such as inflation, liquidity problems etc. This implies that if this is a managed (command) system, then the money supply is perfectly tuned to the production and the "saving" is automatically done by the state by increasing the money supply (quasi-statically) and rationing the output by a central plan. If this is a Smithian world of competition then inflation is so slow as to be negligible and a "fund" of electronic money flows to the "invisible managers" of the Nonhuman potential thus providing for constant and continuous upgrading of the machines without the need for humans to "save money". Thus the "real flux" of goods and services and their "measurement" in *electronic-value-units are* perfectly tuned to each other both systems.

Thus there is no need for humans to save; all their needs are taken care of by their wages. It would also help the modeling greatly if the population were kept constant and if the individuals lived forever just like the machines. However this last assumption is not necessary. It can be shown that stepwise relaxation of the restrictions will not add appreciably to the complexity of the model. Because of the generality of the model it will only be useful as a pedagogical tool --at least in the beginning stages. However it can be shown that the model has many properties of economic models that have been proposed over the last few centuries by various authors. The strength of this model also happens to be its weakness in that because of its generality, the "burden of emulating and explaining reality" has been shifted over to the coefficients of the model--as is usual in all such models of socio-economic systems.

Investment-Consumption (I-C) Equations

In earlier papers Hubey[2], an implicitly probabilistic methodology was used to derive equations which the Investment I(t) and the Consumption C(t) satisfy. The decision process is examined at a segment of time between t and t + Δ, and the function C(t + Δ) related to the function C(t). The equations resulting from this "invariant imbedding" approach, usually accredited to Ambartsumian are

1a) $dC(t)/dt = \beta_{11}(t)\, C(t) + \beta_{12}(t)\, I(t) + \xi_1(I,C,t)$

1b) $dI(t)/dt = \beta_{21}(t)\, C(t) + \beta_{22}(t)\, I(t) + \xi_2(I,C,t)$

As should be noted, the derivation does not purport to be able to give a detailed description of the decision process or the production process. The decisions can be made individually or collectively. Hayek [4], has noted that " *All economic activity is...planning; and in any society in which many people collaborate, this planning, whoever does it, will in some measure have to be based on knowledge... Planning in the specific sense in which the term is used in contemporary controversy necessarily means central planning---direction of the whole economic system according to one unified plan. Competition, on the other hand, means decentralized planning by many separate persons.*"

We thus have a coupled set of differential equations for $I(t)$ and $C(t)$. That the "self-coefficients", i.e. $\beta_{11}(t)$ or $\beta_{22}(t)$ may be positive or negative is clear from the derivation. It can be argued that the "burden of modeling" has been transferred to the coefficients in these equations. This is certainly true since the coefficients are the "intensive" varibles such as the state-of-technology, the consumer preferences which effect the types of goods being produced, and even the laws of the land which can be as capitalistic as laissez-faire, socialistic or "regulated" capitalism. In the applicable cases, the "profit" motive will determine in some ways what is to be produced. In other cases of *societal rationing* of goods and services, the managers of the "means of production" will have to consider quotas, shortages and other things--such seems to be the case where wages and profits cannot be used as an incentive. Also in this model only the effects of investment and Consumption on each other is considered. Furthermore, this system is considered to be a "closed" system in that the effects of other economic systems is not shown in the equations. To consider an "open" economy we would have to consider the above differential equations with "source" or "forcing" terms. (The usage of the word source or forcing seems to come from phsical systems. For example, source is often used in electromagnetic problems since the "source" for electromagnetic fields are charges and currents. In mechanical phenomena, the word "forcing" is used to denote that the system under study is being externally influenced and also due to the fact that the derivations of the equations are via "force" balancing.)

Capital-Labor (K-L) Equations

It is clear that the same type of an imbedding approach can be used to derive the relationship of $K(t)$ and $L(t)$ without taking into account the effects of $I(t)$ and $C(t)$. This has been done in Hubey [2]. These equations are of form

2a) $dK(t)/dt = \alpha_{11}(t)K(t) + \alpha_{12}(t)L(t) + \gamma_1(K,L,t)$

2b) $dL(t)/dt = \alpha_{21}(t)K(t) + \alpha_{22}(t)L(t) + \gamma_2(K,L,t)$

Comments similar to the Investment-Consumption equations can be made about the Capital-Labor equations. Again, only the effects of Labor flux and Capital flux on each other have been modeled by these equations.

Inputs and Outputs Coupled: K-I and C-L

We still need further relationships between the variables as they are still uncoupled. We can get a hint from the usual Capital-Investment (K-I) Equation

3) $dK(t)/dt + \kappa(t)K(t) = I(t)$

This is essentially a cost-definition of capital stocks. It is usally derived via a difference equation formulation, i.e.

4) $K_{n+1} = K_n + I_n$

This equation usually translates somewhat confusingly as the capital "in the n+1 period is its value at the nth period plus the investment during that period". However if the investment did not have an accelerator effect attached to it, there would be no reason to invest except to offset physical depreciation. Hence a more general version is

5) $dK(t)/dt + \kappa_1(t)K(t) = \kappa_2(t)I(t)$

The lhs of the equation displays the inherent behavior of Capital Flux in that it will increase or decrease depending on the value of the coefficient κ_1. That the coefficient should be positive is obvious since without the "source" term on the right hand side, $K(t)$ should decay to zero. The rhs of the equations shows the Investment $I(t)$ in its correct place, as the "source" term for the increase in the contribution to the production-process of

NonHuman Potential i.e. an increase in the Capital Flux. Esthetically it would be much more pleasing to show the increase in knowledge (i.e. scientific-engineering-technological knowHow) as the source for the increase in Capital Flux (and the Labor Flux) as it happens in reality. (Unfortunately this idea is not in vogue at this time since it brings to mind Veblen and his ideas about the "Technocracy". It also evokes images of Marx i.e. "Oh no, not another exploited class" type-comments seem appropriate. The formal solution of equation (5) (with $K(0) = 0$) is given by

6) $$K(t-\tau) = \int_0^t G(t-\tau)\kappa_2(\tau)I(\tau)\,d\tau$$

where $G(t-\tau)$= Green's Function = $\exp\{-\int \kappa_2(\sigma)d\sigma\}$.

The equation above shows explicitly that $K(t)$ will go to zero without $I(t)$. The coefficient $\kappa_2(\tau)$ would seem to indicate that it is an "intensity" function i.e. the knowledge "locked up" or "frozen" in the machines (in Marxian terminology) and "technique" or "technology intensity" in modern terminology. It would seem that if the equations developed here are "measured" in money, then $\kappa_2(\tau) > 1$ would tend to implicitly show a Veblenian way of weighting i.e. it implies that investment is worth more than it costs, hence as alluded to above, equation (5) is not a cost definition of investment. In addition, the Green's function of equation (5) is a "weighting" function of sorts in that the earliest investments are given the least weight and the latest count the heaviest. This integral solution is reminiscent of the Friedman Consumption function (Permanent Income Hypothesis) and was explicitly pointed out in Hubey [2]. This, of course, is more general than the "Ramsey-Solow simplification" which results in equations like $Y(t) = C(t) + dK(t)/dt$. (see Samuelson 6).

The inclusion of education, socialization as well as recreation, and health care, in Consumption C(t)--as was done in Hubey [2] makes it possible to give a very simple expression for the fundamental idea behind the *Human Potential Theory*. An argument, similar to the one above, will lead to an equation for Consumption-Labor (C-L).

7) $dL(t)/dt + \lambda_1(t)L(t) = \lambda_2(t)C(t)$

Thus, the Consumption $C(t)$ acts as a "source" for the increase in the contribution of Human Potential which is commonly called Labor. The solution of equation (7) is similar to equation (5) and it clearly shows that consumption C(t) is the source for the increase in the contributions that humans make toward the production of goods. Since education was included in consumption C(t) in Hubey [2], the "education intensities" or "knowledge intensities" can be shown explicitly in the equation. Thus equation (7) can be thought of as a very simple expression of the *human potential* school of thought.

The set of equations, then are the complete equations of motion for the primary inputs and outputs of an economic system. There is a symmetry in the equations which does not exist in real life. That is, machines do not work or contribute out of their own volition as humans do. However, one can make several observations as to their validity.

First, these are very simple equations and not much more can be done to model a reality as complex as a socio-economic system.. Indeed, it is not known (known in the sense of certainty as in certain physical laws which can be repeated ad infinitum with essentially the same results) that the equations above do indeed model reality. With all due respect to Karl Popper, no matter what happens, the finitely long or finitely many experiments are all that can be performed in reality. The predictive power of a theory is a fundamental determiner in the credibility that the theory evokes in the minds of the literate and the illiterate alike. Even the simplest physical phenomena are more correctly modeled as non-linear phenomena. In that sense, the equations here can only be thought of as linear approximations to reality--assuming of course that the reader can even begin to accept the reasonableness of these equations. Since Economic Theory like other scientific theories and other such human endevears is not completely free of fashion, it would seem that the differential equation models would seem to be pre-empted at the time of this writing by Linear Algebra and Variational schools. An interesting contrast is provided in the works of A. Ll. Wright [10], whose works use continuous time and control-theoretic formulations.

The second observation would be to point out that in this particular time period of rapid advances in computers, --both hardware and software such as Artificial Intelligence techniques and all that it implies such as Artificial Vision, Robotics--notwithstanding the science fiction movies of intelligent robots running amok , there is still the possibility of having machines with almost human-like capabilities--some day !! . This would at least leave open the possibility that the equations developed in this paper which are symmetric with respect to men and machines might one day more correctly model socio-economic systems than they do today.

The Macroscopic State Space

For consistency, the coefficients must be related to each other. The macro-economy has been labeled as a "black box" with the primary "pure" inputs $K(t)$, $L(t)$ and final outputs: $I(t)$ and $C(t)$. The total output $Z(t)$ of the economy would include the intermediate goods $V(t)$, the "transitory form" which the input takes on its way to becoming the final product. Government expenditures, imports and exports are not considered here and can be tacked on to these equations without much conceptual difficulties.

A real economy can be considered in the limit as an economy with an infinite number of goods (It would be more correct to say that a real economy has so many goods that it would be easier to model it as if it had an infinite number of goods) . Adding Eq. (2a) to Eq. (5) and Eq. (2b) to Eq. (7)

8a) $$dK(t)/dt = \psi_{22}(t)K(t) + \psi_{23}(t)L(t) + \psi_{21}(t)I(t)$$

8b) $$dL(t)/dt = \psi_{33}(t)L(t) + \psi_{32}(t)K(t) + \psi_{34}(t)C(t)$$

where the ψs are combinations of the coefficients of the equations derived above. The derivation is straight forward and the details are left out for lack of space. One further relationship needs to be added to the set. Defining $R(t)$ as the *Raw Resources Flux* and $V(t)$ as the System Inventory (Intermediate Goods Accumulation, semi finished goods, circulating capital), we can write a "balance" equation

9) $$R(t) + K(t) + L(t) - (I(t) + C(t)) = dV(t)/dt$$

We can see that on the lhs, the outputs-- Invesment and Consumption goods -- are subtracted from the inputs into the "black box" which is the economic system. If the inputs exceed the outputs of the system there will be an accumulation within the system hence there will be a change in $V(t)$. This is the term that has been called "inchoate wealth" by Hayek. The set of equations can now be written more concisely as a vector differential equation.

10) $$d\mathbf{q}(t)/dt = \mathbf{S}(t)\mathbf{q}(t) + \mathbf{r}(t)$$

where $\mathbf{q}(t)$ and $\mathbf{r}(t)$ are vectors and $\mathbf{S}(t)$ is a "Structural Matrix".

It's a straightforward manipulation to relate the elements of the structural matrix to the coefficients of the previous equations and is omitted here for lack of space. In some way these equations may be considered to be the generalized versions of the Leontieff dynamic equations. Indeed, once the equations are written in this form, a further generalization is simple. Since it is known that "everything affects everything", one might try to use a matrix $\mathbf{S}(t)$ in econometric studies in which none of the coefficients are zero . The total model may be considered to be a two sector model of sorts as done by Shinkai (9) except that the labor and capital flows are not separated here as was done by Shinkai.

Linear Steady State

Without getting into any hair-splitting discussions on the meaning of "steady state" or "equilibrium" one can fiddle with the equations to see what would happen. Steady state is defined as the state at which the state variables namely $I(t)$, $C(t)$, $K(t)$, $V(t)$ and $L(t)$ are "steady" i.e. do not change. In such a case, we are left with a vector equation of the type

11) $$\mathbf{Sq} = -\mathbf{r}$$ where $\mathbf{S}$ is no longer a function of time.

It is certainly logical that the natural resources of a closed system constrain the inputs and the outputs and thus determine the standard of living. Unfortunately, the equation above is probably more accurate over a short period of time rather than long periods. The reasoning is as follows. The physical matter on earth from which all "natural resources" are extracted will not change appreciably over short periods of time--the word short being defined in relation to geological time. Since human society has existed for millions of years (and hopefully will exist for millions more), a time span of less than a century seems short by

comparison. During this "short period of time", the only thing that can change the natural resources of a closed system will be a "scientific invention" which makes things valuable that were considered valueless before the invention. Similarly, things that were scarce and valuable might become unnecessary once substitutes are invented. A modern example that comes to mind is the use of optical fiber for copper. In the near future, hydrogen (extractible from water) might make fossil fuels unneccessary. If the aggregate state variables are broken down into different sectors, it can be seen that the equations above -- equations (10) and (11) are similar, at least in spirit, to the linear models due to Leontieff, Von Neumann, and Arrow.

Linear Extensions and Other Theories

So far only the simplest linear cases have been investigated without causing too much difficulty. It can now be shown that with minor modifications a more realistic model of a real economy can be obtained. For example, it was mentioned in Hubey (1) that in order to make a symmetric argument education and training was included in Consumption $C(t)$ and not as an Investment , $I(t)$. This was justified on the basis of making a functional aggregation rather than one based on time. In most writings on Economic Theory , education is included as an Investment since the service is not considered to be consumed immediately but is a kind of a "saving" for the future. In the models developed here and earlier, a good is "tagged' as an investment or consumption good depending on end-use. Therefore it would be more realistic to realize that in equations (1) and (7) some of the effects of $C(t)$ (the part that includes education) will be realized after a time delay. Thus the effects of $C(t)$ on $L(t)$ will be delayed by a time period τ. We should then use $C(t-\tau)$ to account for the effect of eduction on $L(t)$. In order to show the effects of such consumption we can use

12) $$\int C(t-\tau)d\tau$$

for the source term. However, a more correct (and more difficult to treat analytically) version would be weighted over time. i.e.

13) $$\int W(\tau)C(t-\tau)d\tau$$

where a reasonable choice might be $W(\tau) = \exp(-\zeta\tau)$. Since delay-differential equations (or equations with convolution integral type source terms) cannot be readily solved we can make a *binary simplification* where we split $C(t)$ into two terms ; those whose effect is immediate and those whose effect shows up "later" i.e. use delay terms for the educational component of consumption

14) $$C(t-\tau) = C(t) - \tau\, dC(t)/dt$$

This aproximation still leaves the equtions as linear and reasonably tractable. Substituting the result for $C(t-\tau)$ --from equation (14)--- for $C(t)$ in equation (7) and substituting for $dC(t)/dt$ will give an equation for $dL(t)/dt$ in terms of $C(t)$ and $I(t)$. Solving for $dC(t)/dt$, and substituting for $dL(t)/dt$ we will obtain an equation for $dC(t)/dt$ in terms of $L(t)$ and $K(t)$. I.e we can obtain equations of type

15a) $$dL(t)/dt = F(C(t), I(t))$$
15b) $$dC(t)/dt = H(L(t), K(t))$$

One can use a delayed function for investment $I(t-\tau)$ to show the effect of investment in research machines (of the type now employed such as linear accelerators, and research in fusion etc). Unquestionably the future holds space for much more knowledge workers workers employing much more machines. Similar to education this type of investment's effect on $K(t)$ will show up later and not immediately. Thus it is relatively simple to justify the use of a linear vector equation of type given in equations (10) and (11) in which very few of the elements of the matrix are zero. It would seem intuitively that an econometric analysis of this type should not make any of the elements of the $\mathbf{S}$ matrix zero.

A Nonlinear Model and Production Functions

Instead of the linear equation for K- L we can use a non-linear version

16a) $dK(t)/dt = a_1 K(t) - (a_1/b_1)K^2(t) - (a_1 a_3/b_1) K(t) L(t)$

16b) $dL(t)/dt = a_2 L(t) - (a_2/b_2) L^2(t) - (a_2 b_3/b_2) K(t) L(t)$

These equations can be derived by assuming that Capital $K(t)$ and $L(t)$ are competing for the same limited supply of funds. The equations are of the type derived by Volterra for competition between two species of animals, but their properties make them attractive and they are an extension of the linear theory. The Lotka-Volterra type equations have been used in economic theory by Samuelson [7,8], Goodwin [1] and analyzed by Medio [5] from a socialist point of view. It can be shown that the solutions of the above set approach the steady state

17a) $K = (b_1 - a_3 b_2)/(1 - a_3 b_3)$

17b) $L = (b_2 - b_3 b_1)/(1 - a_3 b_3)$

for $b_1/a_3 > b_2$ and $b_2 > b_3$, $L(t_0) > 0$ and $K(t_0) > 0$

Furthermore, equating $dL(t)/dt$ from Eq. (7) and Eq. (16b) we obtain

18) $C(t) = (1/\lambda_2)\{(a_2 + \lambda_1) L(t) - (a_2/b_2) L^2(t) - (a_2/b_2) b_3 K(t) L(t)\}$

Substituting into equation (1b) and solving, we obtain

19) $I(t) = \int G_1(t-x) \{ A_0(x)L(x) - A_1(x)L^2(x) - A_2(x)K(x)L(x)\} dx$

where the $G_1(t-x)$ is the Green's Function of the linear differential equation for $I(t)$ in equation (1b) and

20a) $A_0(x) = \beta_{21}(a_2 + \kappa_1)/\kappa_2$

20b) $A_1(x) = \beta_{21} a_2/(\kappa_2 b_2)$

20c) $A_2(x) = \beta_{21} a_2 b_3/(\kappa_2 b_2)$

With a similar set of operations we can obtain relations for $C(t)$ as in equation (19). Recalling that

21) $Y(t) = I(t) + C(t)$

it is clear that equation (19) and the one for consumption --not shown here-- will constitute a dynamic production functional. It can be shown that for certain values of the coefficients, the conditions $\partial Y/\partial K > 0$ and $\partial^2 K/\partial K^2 < 0$ can be obtained. Similar results can be shown for $\partial Y/\partial L$ and $\partial^2 Y/\partial L^2$. Similar calculations for the linear set of equations will show that $\partial^2 Y/\partial L^2 = \partial^2 K/\partial K^2 = 0$. It should be pointed out that the production function thus derived in dynamic in the sense that it gives the output over time for inputs into the economic system over time. Since the Green's function is for a first order differential equation it will be of exponentially decaying (or increasing depending on the coefficients) form. Hence the Green's function is a "weighting function" in the sense that it gives the output at time t for a unit input at time x. In this sense it a mathematical representation of the production process as envisioned by Hayek. The I-C equations can be also non-linearized to give an Input Functional (a la Hayek) but the solution of the resulting nonlinear set for $I(t)$, $C(t)$, $K(t)$ and $L(t)$ would be a formidable problem.

Summary

It has been deemed useful to split the final output $Y(t)$ in the simplest manner, into Consumption $C(t)$ which is for the immediate satiation and sustenance of the people and Investment $I(t)$, which is the good and services which must be "resunk" into the economy. In a sense one can think of $I(t)$ as "consumption" for the stock of Non-Human Potential (Capital), as $C(t)$ is "nourishment" for the Human Potential $H(t)$. The investment includes "subsistence level" nourishment (i.e. parts and "repair service" much as medical care) as well as new equipment embodying new engineering-scientific knowledge which can increase the contribution flux $K(t)$ of $N(t)$. Quite symmetically Consumption $C(t)$ includes "formal education" and "training" for human beings to enable them to live as contributing

and productive members of society. It should be pointed out that this point of view, namely that education is not an "investment duty" to be endured but a "pleasure" to be consumed is in a sense isomorphic to the new humanists' view as it is identical with the Greek view of antiquity (i.e. School = leisure). There is no way, at this point, to show explicitly the effect of the generation and creation of scientific knowledge on the standard of living of a society. A model which intends to take into account the creation of knowledge should consider such knowledge as a specific and special "form of wealth" much like energy and raw materials. It is this special form of wealth that allows all other raw materials to be regarded as "resources". This "transformation" type of problem should probably be treated with thermodynamics type of formalism.

Other results such as "steady state" growth, unsteady growth, oscillatory growth patterns can be derived from these equations. It has already been shown how steady state solutions can be obtained from the model presented here and elsewhere [2,3]. It has also been shown that a particular non-linear version of the set of equations, has desirable "production function" properties.

Unusual terminology and unfashionable mathematics are used in the derivation of the models in this paper and the others before it. Some of the reasons for the use of unusual terminology is that , although seeming pointless it is hoped that these terms will not evoke in the readers' minds the connotations with which these terms are stigmatized. The terms Potential and Flux have been used because they are still unstigmatized in economics. This paper and others before it (and others to follow) are being written in the spirit of Hayek (and more modernly "glasnost" :-)).

References

1. Goodwin, R.M., "A Growth Cycle", in Socialism, Capitalism and Economic Growth, C.H. Feinstein, ed, Cambridge University Press, Cambridge
2. Hubey, H.M. and Chichester, F.D., "*A Linear Differential Equation Model for Investment and Consumption*", Proceedings of the Nineteenth Annual Pittsburgh Conference on Modeling and Simulation, May 5-6, 1988.
3. Hubey, H.M. and Chichester, F.D., "*The Macro Economy: Coupled Linear Differential Equations for Capital and Labor Inputs*", Proceedings of the Nineteenth Annual Pittsburgh Conference on Modeling and Simulation, May 5-6, 1988.
4. Nishiyama, Chiaki and Kurt R. Leube, Ed., " The Use of Knowledge in Society", p. 213 ,in The Essence of Hayek, Hoover Institution Press, Stanford, California
5. Medio, Alfredo, "A classical model of business cycles", in Growth, Profits and Property, edited by Nell, Edward J., Cambridge University Press, Caambridge
6. Samuelson, Paul, "Pasinetti Paradox in Neoclassical and General Models", in Collected Scientific Papers Vol 3 p.189
7. Samuelson, Paul, "Generalized Predator-Prey Oscillations in Ecological and Economic Equilibrium" in Collected Papers, Vol 3.
8. Samuelson, Paul, "A Universal Cycle ?", in Collected Papers, Vol 3.
9. Shinkai, Y. "On Equilibrium Growth of Capital and Labor" , Int. Econ. Rev. Vol.1, p107-111
10. Wright, A.Ll. An Introduction to the Theory of Dynamic Economics, Clarendon Press, Oxford, 1984