PSYCHO-SOCIO-ECONOMIC EVOLUTION OF HUMAN SYSTEMS

Almost all mathematical economics theory revolves around measurements in monetary units. Other factors which are admittedly important are assignsd to the intangibles heap, or only verbal explanations, or descciptions offered. The few psychometric measurements that exist in the social sciences are either under attack, are not deemed germane to economics or their measurement space is [0, 100] or some other finite interval rather than the $[-\infty, \infty]$ or $[0, \infty]$ space of most natural mathematical sciences, such as physics. In the physical sciences there exist fundamental dimensions in which these properties are measured, with the appropriate instruments and in appropriate units: for example Distance (far-close), Time (early-late), Temperature (hot-cold).

Other properties such as traits or characteristics which we attribute to people, such as selfishness-altruism dfsciplin--laxnats, craativity- dullness, do not seem to be measurable If we can imagine a battery of tests (the instrument) to measure these propartias, we should assign to them values in a range, say [0,1] or [0,100]. Ordinary language, thens works actually with two scales moving in opposite directions with the adjectives ar increments in thfs one-dimensional space. For concreteness, let us try to define the quality/quantity, or the magnitude or the amount of conmunism C(x) in a system (i.e. nation). Assuming that the necessary tests exist, we can than measure the magnitude of communism. If we define its opposite P(x) as privatism, with all that if might entail (i.e. no-public schools, competition, no public police force, no taxes etc.) then it is clear that they should obey

$$C(x) + P(x)=1$$

We have chosen the interval to be [0,1] since any other finite interval can be obtained from it. Equation (1) means that if we assign to a nation the value C(x)=1 then it must be perfectly communist and therefore perfectly non-privatist i.e. P(x) I O. Since it is always easier to work with variables in the unrestricted ra"Qe [-oo , a] it Will be easier to develop theories in this natural space. We can always transform to the [0,1) space by defining a scaling Function $\sigma(x)$, such that

$$P(x) = \int_{-\infty}^{x} \sigma(y)dy \quad \text{and} \quad \int_{-\infty}^{\infty} \sigma(y)dy = 1$$

By also defining

$$C(x) = \int_{x}^{\infty} \sigma(y)dy$$

We can see that the condition of Equation (1) is satisfied. Since we can always revert to the laboratory space (the usual Psychometric or econometric space e.g. I.Q., personality , discipline, creativity) as they are defined by equations similar to Equations (5) and (6), we can extend the concept by suitably defining P(x) to be a joint function of the relevant coordinates.

We just define a function $\sigma(x_1 x_2 \ldots x_n)$ such that

$$\int_{-\infty}^{\infty} dy_1 \int_{-\infty}^{\infty} dy_2 \ldots \int_{-\infty}^{\infty} \sigma(y_1 y_2 \ldots y_n) dy_n = 1$$

so that

$$P(x_1 x_2 \ldots x_n) = \int_{-\infty}^{x_1} dy_1 \int_{-\infty}^{x_2} dy_2 \ldots \int_{-\infty}^{x} \sigma(y_1 y_2 \ldots y_n) dy_n$$

where the x_i are defined to be the fundamental dimensions (a basis) of human characteristics. The characteristics of an ensemble of individuals can be obtained as a weighted average of the ensemble can be represented as a point in this behavior space (determined by the x_i).

EVOLUTION OF TRAITS IN CHARACTERISTICS SPACE

An individual's character will be shaped by the events of his/her life. Although the events of his/her life will be mostly through interactions with people, it will actually be tha characteristics or the traits of the people that will affect him/her. Thus someone who has experienced cruelty might develop this same trait. An act of kindness might cancel an act of cruelty. The same experience at different times or stages of development might give rise to different dispositions or outlooks on life.

Assuming necessary dynamic mathematical medals of development of particular traits or characteristics of individuals have been constructed by psychologists, economists would still be left with the problem of generalizing from the individual to the society as a whole. This is not a matter of straight foward mathematical extrapolation. The equations for the characterization of individuals in behavior space must necessarily contain random parameters. Even for the linear equations, such random fluctuations in the coefficients result in equations for the average which are different than the original equation.

Since the generalization from the micro to macro economic point of view, of necessity, must be based upon some kind of average over the ensemble of individuals, the behavioral equations of macroeconomics willbe different than those based on the deterministic behavior of single individual. Thus, the model of the development of the individual (motion in time through the space of attributes) should, in general, have a non- homogeneous term (forcing or source term) and random initial conditions. Random differential equations can be visualized as representing families of deterministic sample differential equations. The ultimate goal in the solution of random differential equations is the complete statistical description of the output from the statistical knowledge of the coefficients, the initial conditions and the output. Often, however the determination of a limited amount of information about the solution process such as expectation, correlation function or spectral density is sufficient. From a mathematical as well as a physical point of view, the characterization of stochastic differential equations is strongly dependent upon the manner in which the randomness enters the equations. It is convenient to distinguish three basic types of stochastic differential equations (Syski,1967).

1) Random Initial or Boundary Conditions
2) Random Forcing
3) Random Coefficients

The first two cases are fundamentally simpler than the case of random coefficients because of the deterministic relationship of the statistical properties of the solution to the statistical properties of the elements of randomness. The general random non-linear differential equation can be represented quite generally in form:

$$\frac{dx_\mu}{dt} = F_\mu(x_1 x_2 \ldots x_n, t;\omega) \qquad (\mu = 1, 2, \ldots, n)$$

Since any mathematical object that depends on a random variable ω is itself random, Equation (8), together with the initial value $x_\mu(0)= c_\mu$ determines a stochastic process $x_\mu(t;\omega)$ providing that for each individual $\omega \in \Omega$ the equation has a unique solution. Representing x in n-dimensional space as a point, Equation (8) then gives velocity of motion of the individual through this state space. From each initial point c_μ, (for each value of ω, treated as a fixed parameter) we get a trajectory which is the solution of Equation (8). An ensemble of initial points moving like a compressible fluid according to Equation (8) will have its density $\rho(x,t)$ obey the number conservation or the continuity equation given by (Van Kampen, 1976).

$$\frac{\partial}{\partial t}\rho(x, t) = -\sum_{\mu=1}^{n} \frac{\partial}{\partial x_\mu}[F_\mu(x_1 x_2 \ldots x_n, t;\omega)\rho(x, t)]$$

Since F(x,t:o) is a stochastic process, we expect the solution of Equation (9) also to be a stochastic process. However, through the lemma of van Kampen (1967), it turns out that the average of the density of points $<\rho(x,t)>$, is identical with the probability density p(x,a) of x(t), arising from the randomness of the coefficients and the initial conditions of Equation (8). Thus, the non-linear problem of Eq. (8) has been turned into a linear one. One can obtain a master equation for $<\rho(x,t)> = p(x,t)$ of form

$$\frac{\partial}{\partial t}\rho(x, t) = K(x)\rho(x, t)$$

where K(x) is an operator acting on the x-dependence of $\rho(x,t)$, but not on its (time) t-dependenca. Knowing the probability density of the motion of an ensemble of individuals in attribute space with a given initial distribution is equivalent to a characteritation of a system from the averaged behavior of its constituent elements.

EQUATIONS FOR THE AVERAGES

In some cases it is possible to get solutions by different methods. For the differential equation

$$\frac{dx_\mu}{dt} = f_\mu(x_\mu, t) + G(x_\mu, t)\omega(t) \qquad (\mu = 1, 2, \ldots, n)$$

where x and f are n-vectors, G is an $n \times m$ matrix and o is an m-vector zero-mean white Gaussian noise with

$$\langle \omega(t)\omega(\xi) \rangle = Q(t)\delta(t-\xi)$$

where Q(t) is positive semidefinite and $\delta(t-\xi)$ is the Dirac delta function, one can obtain an equation for the first-order and second order probability density functions, as below

$$\frac{\partial}{\partial t}\rho(x, t) = -\sum_{\mu=1}^{n} \frac{\partial}{\partial x_\mu}[p(x, t) f_\mu(x_\mu, t)] + \left(\frac{1}{2}\right) \sum_{\mu, \upsilon=1}^{n} \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial x_\upsilon}[pGQG]_{\mu\nu}$$

From the equation above, known as the Kolmogorov forward equation or the Fokker-Planck equation (Jazwinski, 1970) one can then find the average characteristics, $<x_\mu>$. For the linear case,

$$\frac{dx_\mu}{dt} = F(t)x_\mu + G(t)\omega(t)$$

the Fokker-Planck equation reduces to simpler form. However, in this case, it is possible to find ordinary differential equations which the mean $<x_\mu>$ and the covariance matrix P(t) must obey. These are given by

$$\frac{d}{dt}\langle x_\mu \rangle = F(t)\langle x_\mu \rangle$$

$$\frac{d}{dt}P(t) = F(t)P(t) + P(t)F^T(t)G(t)Q(t)G^T(t)$$

For a more general case than Eq. ( )

$$\frac{dx_\mu}{dt} = F(t)x_\mu + \alpha G(t;\omega)x_\mu$$

where α is a parameter determining the size of the random fluctuations and is not restricted to being Gaussian white noise, it is still possible to get an approximate integro-differential equation fro the average alone. For the special case of F(t)=constant, it is shown (Van Kampen, 1976) that the average $<x_\mu>$ approximately obeys an integro-differential equation

$$\frac{d}{dt}\langle x_\mu \rangle = F(t)\langle x_\mu \rangle + \alpha^2 \int_0^\infty d\zeta \langle G(t) e^{F\zeta} G(t-\zeta) e^{-F\zeta} \rangle \langle x_\mu \rangle$$

There are other methods of obtaining the above equation. What is important in the descriptions of systems of form Eq. () or () or () is that the evolution of an individual or the ensemble has a deterministic component F(t) or F(x,t) which is or can be due to bio-genetic effects (for populations) or technological-scientific constraints (for societal evolution), and also unexplained deviations from this path due to randomness inherent in $G(x_\mu,t)$. The deterministic trajectory is, in effect, the mathematical prescription of the often heard refrain 'we are all alike' and the random fluctuation is what makes us all 'unique individuals' Hence each individual is allowed to wander in the (psycho-socio -economic) state space and plot out a unique trajectory within the bounds and constraints imposed by the differential equations ( ), ( ), or ( ), but the manifest destiny of humanity will be given by the averaged equations (5) or ( ). The randomness of the initial conditions extends the idea of evolution of an individual to the development of the society as a whole (or the evolution in the mean of a nation). Of course, the next logical step would be to create a system of socio-economic systems (i. e. the community of nations), each with a given level of socio-economic development. The averaged equations describing a given system would again be treated as an ensemble with random initial conditions and random coefficients to model the world economy. We would expect such boot-strapped (iterative-recursive) systems to behave quite differently than the original microscopic equations from which they were derived. Hence, it would be possible to include such variables in macro-economic theory as dis cipl ine of the work force or entrepreneurship/innovativity of management without being rebuked for generalizing about national characteristics.

AN EXAMPLE: CONSERVATISM - LIBERALISM CYCLES

A simple example to illustrate some of the foregoing might be to model a recurring psyche-social process such as the oscillations in political or religious beliefs. It has been observed that church attendence waxes and wanes and there are oscillationsin the political beliefs between conservatism and liberalism. Such oscillations have also been observed in fashions, hairstyles, etc. These processes can be modeled most simply by the classical harmonic oscillator equation

$$\frac{d^2 z}{dt^2} + \omega^2 z = f(t)$$

The homogeneous solution is given by

$$z(t) = b_1 \sin(\omega t) + b_2 \cos(\omega t)$$

while the nonhomogeneous solution is

$$z(t) = L^{-1}(t) f(t) = \int_0^t G(t-x) f(x) dx$$

where the Green's function G(t–x) is given by

$$G(t-x) = \frac{\sin(\omega(t-x))}{\omega}$$

We now allow random fluctuations in the frequency

$$\omega^2 = 1 + \alpha\xi(t)$$

where $\xi(t)$ is a stochastic process with zero mean and correlation time γ_c. This implies that the individuals comprising society are out of phase with each other by a random amount. We should note in passing that great social movements occur when whole populations can be made to move in phase, not out of phase. Application of the results of Eq. ( ) results in (Van Kampen, 1976)

$$\frac{d}{dt}\langle x_\mu\rangle = F(t)\langle x_\mu\rangle + \alpha^2\int_0^\infty d\xi\langle\xi(t)\xi(t-\zeta)\rangle(G(t)e^{F\zeta}G(t-\xi)e^{-F\zeta})\langle x_\mu\rangle$$

with $F = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $G = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

The result is

$$\frac{d^2}{dt^2}\langle z\rangle + \left(\frac{\alpha^2 c_2}{2}\right)\frac{d}{dt}\langle z\rangle + \left(1 - \frac{\alpha^2 c_1}{2}\right)\langle z\rangle = 0$$

where

$$c_1 = \int\langle\xi(t)\xi(t-\zeta)\rangle\sin(2\zeta)d\zeta \text{ and } c_2 = \int\langle\xi(t)\xi(t-\zeta)\rangle(1-\cos(2\zeta))d\zeta$$

Thus a randomness in the frequency causes a damping of the average amplitude. Eq. ( ) is characteristic of processes with negative feedback. If Eq. ( ) theoretically explained the oscillation in the cultural-political outlook of individuals of a society, one would expect wild oscillations in the amplitude (resonance phenomenon), as has happened in the past in societies in which the people's outlook was rigidly controlled and directed from the top. With the random fluctuation in the natural frequency, as one would expect to happen in an open society which tolerates pluralism, the equation for the average <z> becomes damped thus implying a learning process during which the disturbances decay. The amplitude tends toward the central-beliefs held by the population, indicating a fundamental stability of society.

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