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William G. Vogt
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THE MACRO ECONOMY : COUPLED LINEAR DIFFERENTIAL EQUATIONS FOR THE CAPITAL AND LABOR INPUTS

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ABSTRACT

The primary inputs into the economy--labor and capital flows --emanating from the primary agents, Human and Non-Human sources is represented by a coupled set of ordinary differential equations.

Key Words: Macro-Economics, Primary Economic Inputs, Differential Equations,

INTRODUCTION: The Problem of Taxonomy:

In the usual definitions of Economic Theory, the criterion for whether a good is to be considered an investment or consumption is based upon temporal considerations. Hence, grain that will be saved for sowing next year's crop is considered an investment and that portion of it which will be used to make beer and the portion of it which will be made into flour and thence bread is considered "circulating capital". Only that portion of it when it is finally made into bread is considered to be a consumption good. Of course the part of it that gets fed to animals might be considered either as investment good, consumption good or circulating capital depending on to which use the animal is put. For example if the animal eating this good is to be butchered then the grain might be considered circulating capital. If the animal is used as a pack animal, then the animal is being treated as a replacement for machinery, hence the grain will probably be considered as an investment good. One need not deal with animals or with food to run into similar problems in classification.

There are other, probably equally valid, ways of categorizing the output --as well as the inputs--- of an economic system. One can consider the final products of the economic system and the primary inputs into the system only --labor and capital "flows" -- and leave the complexities of the middle stages--both in space and time---out of the macro economic model. The models presented here as well as in Hubey & Chichester [1] are phenomenological models of this type. Only the relationships of the primary inputs to each other are considered in this paper. The models for the primary final outputs are dealt with in Hubey & Chichester[1].

The models in this paper and in [1] use slightly different definitions than as is common in economic theory. To minimize the connotations that readers might attribute to commonly used words, in this paper we would like to refer to what are normally called Capital Stocks --actually only a part of it, as will be clearer later---and Labor as "Non-Human Potential" and "Human Potential". The word potential is used in a sense similar to the way it is used in Physics. It is preferable to use this term instead of "stocks" since stocks are usually depleted in the course of normal economic activity whereas human potential i.e. the knowledge and the ability to contribute to economic processes are usually passed on to the next generation in the form of an educated populace. Also we call the input emanating from humans and machinery "flux" as in "magnetic flux" since fluxes are caused by "potentials" and do not dissipate or get depleted as long the potential is there. Also, for verbally esthetic reasons , the phrase "human potential" is much more preferable to " a stock of humans" or "labor stocks" !

We consider that there are essentially two active agents ---humans and machinery -- that combine symbiotically to produce goods. Of course machines do not really participate in this economic process through their own volition. However the inputs (i.e the fluxes) of both Human and Non-Human Potentials do behave when modeled mathematically as if they are intelligent and complementary (in most cases) to each other. It is true that for short periods of time, machines will replace human activities(i.e unskilled labor) . however since new generations are always (almost or at least desirably so) more educated (i.e exhibiting more technological skills) than the previous generations, they are always being trained to perform activities which new machines cannot yet perform. So the problem of temporal definitions cloud the issues as before Especially now, as machines become more and more intelligent --because of the extensive "imbedding" of digital computers and Artificial Intelligence techniques in them -- the necessity of "technological literacy" will become more pronounced It will be even more difficult for human societies to "stay ahead" of machines.

So, in keeping with the spirit of the definitions above, in these papers "investment" is defined analogously to "consumption" in that it is treated as "nourishment" for Non-Human Potential and thus its usage is symmetric with the usual definition of consumption. This includes repairs, maintenance and new techniques imbedded either in new machines or as add-ons to existing machines. This definition assumes that an abstract "lumped Non-Human Potential" exists whose flux can be increased by adding more "physical" or "intellectual" ability to it. In short Non-Human Potential gets larger only if its flux can be made to increase. It is a simple fact of physics that it takes energy to move matter in three dimensional space which we inhabit and goods cannot be produced without some agent(s) acting upon matter. Living beings get their energy from food and Non-Human Potential also needs energy to do work and thus needs "to be *nourished*". For example oil or coal stocks do not get consumed by humans but are used by machines. We might say that the machinery "*consumes*" this "*nourishment*" as humans might consume food. Although meant to be taken in a figurative way, the words "consume" and "nourish" are very apt descriptions of this process of doing work and creating useful objects which we call goods.

Similarly, Human Potential can be increased by a variety of means such as better diets, health maintenance and of course by education. This last point, of course, implies that education is "consumption" and not "investment" and is only marginally treated in economic models which deal with consumption and investment. Again, since the taxonomy used here is not temporal but purely functional there is no other choice. The taxonomy used here also seems to imply that investment and consumption can only be a part of what is normally called the Final Output of the economy, since it is only at that time that the output can really be tagged as to its function.

There is also the problem of "measuring" the "value" of the fluxes. One can imagine that the fluxes are measured as product of an "extensive parameter" such as the "physical energy" expended and an "intensive parameter". This "intensive parameter" must have the dimensions of "knowledge"-- what is usually called "technology" , "technique" or "skill". Useful work can only be done by expending "physical energy" in accordance with a very strict prescription. This "orderly physical expenditure of energy" to move things, or otherwise impact or influence the physical world is what produces goods. It is this set of "prescriptions" that is called "knowledge" in this paper. Although "order" and "entropy" and probability theory are inextricably intertwined in the Physical and Mathematical Sciences, most usage of entropy in papers in Economic Theory deals with only "entropy" in the physical sense i.e. increase of the entropy of the universe and heat death. Brillouin {2} seems to have been the one of the first to entertain the thought of connecting "negentropy" --negative entropy--with knowledge and order. He attributes this idea to Schrodinger. Of course in later developments, entropy became firmly entrenched in Information Theory and thus Computer Science and Communications Theory {3}. Georgescu-Rogen has done an excellent job of collecting all thoughts on entropy and its various shades of meanings in both the physical and economics literature {4}. However none of this has yet made its way into economic theory in any reasonable way--- for example "knowledge" being a type of wealth which makes possible the extraction of wealth from common natural and raw materials of the universe. In this way "knowledge" in "living systems" behaves similarly to entropy in physical systems in that it is always increasing.

The measurement of "knowledge" --and its relation to information and entropy --of course is beyond the scope of this paper and indeed beyond the present state of economic theory and cannot be treated here. However, we can use this idea to classify the fluxes into the economy. It will probably be best to define knowledge intensity as a continuous function to simplify the derivation of the model equations. We can always discretize this function and break it up into the usual classes as used in economic theory i.e. unskilled, skilled etc.

CAPITAL AND LABOR EQUATIONS

The primary agents in the economic production system are the "flux" from Human Potential i.e. Labor and the "flux" from Non-human Potential. The decisions regarding the shares of the inputs into the economy will for the most part be determined by the state of technology and the existing

labor force and its skills. Thus an approach similar to the one used in Hubey & Chichester {1} ca[n] be used to develop the equations listed below

$$d/dt\ K(t) = \alpha_{11}(t)K(t) + \alpha_{12}(t)\,L(t) + g_1(K,L,t) \qquad (1\ a)$$
$$d/dt\ L(t) = \alpha_{21}(t)K(t) + \alpha_{22}(t)\,L(t) + g_2(K,L,t) \qquad (1\ b)$$

As in {1} only the linear case will be treated in any detail The equations will make more physic[al] sense if observed in the integral form Integration of Equation (1 a) (with K(0) =0) results in

$$K(t) = \alpha_{11}(x)K(x)\ dx + \alpha_{12}(x)\,L(x)\ dx \qquad (2)$$

The first term on the rhs is that component of labor-flux L(t) which has gone into the production o[f] Non-Human Potential (i.e. capital stocks that actively contribute to creating goods). The integra[l] runs from time zero to the present time t so that all the labor that has gone into the creation of thi[s] potential is summed up. The second term is the net contribution of K(t) throughout the same tim[e] period; that is it is the acceleration effect of the use of K(t) to increase the Non-Human Potentia[l] which results in an increase in the flux i.e. K(t). One can also attribute meanings to the coefficient[s] in the equation. The "wear" and depreciation may be considered to be included in the second term Thus this equation may be considered to be consistent with the "labor theory" of value. It is th[e] coefficients in the equation that will determine just how much of the knowledge (i.e. scientifi[c] knowledge) is "embedded" in the Potential. The Non-Human Potential could have (at all times) "embedded" in it the sum of knowledge that has been created. One would say that in such [a] system, scientific knowledge would quickly be transformed into useful technology. This is jus[t] another way of saying that the machines are assumed to be state-of-the-art. In other societie[s] technological changes might not keep abreast of scientific inventions and discoveries.

A similar line of reasoning may be pursued to explain the terms of the other equation. The increas[e] in the contribution of labor, resulting in greater output, comes about because of the creation o[f] knowledge (and its transmission among the population) and its resultant "embedding" in physica[l] "stocks" (i.e. Non-human Potential). Thus the rate of change of the labor flux is proportional to th[e] scientific-engineering knowledge which manifests itself in the capital flux, so that

$$d/dt\ L(t) = \alpha_{12}(t)\ K(t) \qquad (3)$$

is, in effect, another expression of the labor theory of value. The other term, shows the direct effe[ct] of that portion of labor flux which is devoted to the creation of a store of knowledge and subsequent transmission to the later generations.

SUMMARY

The models presented in this paper and in Hubey & Chichester {1} are phenomenological and try t[o] abstract only the most important characteristics of a simple generic economic system. Non-linea[r] models of Capital and Labor inputs can be developed using other arguments, as can be seen i[n] Hubey {5}. It would be foolish to claim that these simple models can be used to successfully predi[ct] or forecast the behavior of any real economic system. However, in the least, they can be used a[s] pedagogical aids.

Furthermore, these equations are unique in that the most elusive and most important of all econom[ic] variables --i.e. knowledge-- is taken into consideration as an integral part of the models. Also, mo[st] macro-economic models are discrete models in contrast to reality in which the economic processe[s] take place continuously in time. The models here, have deflected the burden of econom[ic] explanation to the "intensive" variables --i.e. the coefficients. Since the derivation did not at a[ll] make use of any conventional money-oriented measures, the models --as simple as they may be are still applicable for any type of system. They are similar to attempts in the field of Artifici[al] Intelligence in which a limited amount of intelligence is given to a computer which in effect become[s] an expert in a "toy domain".

Since money was not considered at all, the effects of interest rate, money supply and inflation et[c.] cannot be treated explicitly. However to do that one would have to consider a capitalist economy at least a marketized economy even if nominally socialist---and not a generic one. In any cas[e]

... been implicitly taken into account but even other variables such as the "skill level" of the work force and the "discipline of the work force" can also be accounted by properly "tweaking" the appropriate "intensive" variables. Indeed "skill levels" and "worker discipline" etc are intensive properties of economic systems. Thus these models are only a first step toward the development of more sophisticated models in which the "knowlege distribution" of the system can be used to account for such topics as unemployment, "technology transfer", and the influence of "education" on the system.

These models can also be extended to include government and the effects of the "growth of money" and interest rates. But of course these will be special cases of economies in which the governments only indirectly influence the economy through fiscal and monetary measures --i.e "fiddling" with the rate of growth of money and government spending such as for defense, public works and support of scientific research through grants.

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LINEAR DIFFERENTIAL EQUATION MODELS FOR INVESTMENT AND CONSUMPTION

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ABSTRACT

Models are developed to show the interdependence and relationships of the two primary final products of a simple "generic" economy in which there is no government. Slightly different conventions are used in the definitions of investment and consumption which results in symmetric treatments of both outputs.

Key Words: Macro-Economics, Primary Economic Outputs, Coupled Differential Equations

INTRODUCTION

An economic system is too complex to be modeled exactly or in fine detail. In order to make headway one has to abstract only some of the most important attributes of the system. In this paper and in Hubey & Chichester{1} mathematical models are created to represent the interaction and behavior of the primary inputs and outputs of a simple generic economic system. In both papers the government does not exist. Whether one considers this system to be guided by the "invisible hand" or the "disappeared state" is only a matter of personal taste. In this paper only the final products of the economic system are considered leaving the complexities of the middle stages--both in space and time---out of the macro-economic model. In some ways, this approach is similar to Thermal (Statistical) Physics (Thermodynamics by its old name and Information Theory by its modern name), where the end states of "processes" are taken into account only and not the paths that these processes trace out in Thermodynamic Phase Space. In this paper a hybrid approach is taken. This paper uses some specific and uncommon terminology and thus the reader is urged to read Hubey & Chichester {1}.

INVESTMENT-CONSUMPTION EQUATIONS

If we denote Investment by $I(t)$ and Consumption by $C(t)$ we can derive equations for the relationship of these variables by examining the process by which these final products are created. We examine the economic system at a small segment of time between t and $t+\Delta t$ and try to relate the function $C(t)$ to $C(t+\Delta t)$. To this end we define a function $\sigma(t)$ such that $\sigma(t) + \vartheta(\Delta)$ gives the probability that a decision will be made --somewhere in the economic system by someone-- regarding the split of the final output of the economic system into $C(t)$ and $I(t)$. The notation $\vartheta(\Delta)$ (Order of Δ) denotes a function such that $\vartheta(\Delta)\to 0$ faster than Δ as $\Delta\to 0$.

Now, a certain amount of consumption goods will be produced at time $t+\Delta$ without having a decision made (at that interval of time) regarding the production of these goods. This is true in all types of economic systems since the important decisions, whether made by a centralized management (some people might call them "command systems") or by the management of independent and decentralized enterprises, are made some time in advance according to some plan which takes into account the market forces. These decisions will for the most remain in effect for much longer period of time than the period in consideration. These consumption goods might be called "autonomous" production by some authors. The probability of this happening, from the definition of $\sigma(t)$ is $(1-\sigma(t)\Delta + \vartheta(\Delta))$ i..e we assume that the probability of occurrence of this in Δ is proportional to Δ . Hence the expected amount of such goods contributing to $C(t+\Delta)$ is

$$[(1-\sigma(t)\Delta + \vartheta(\Delta)]C(t) = (1-\sigma(t)\Delta)C(t) + \vartheta(\Delta) \quad (1)$$

It is also possible that at any given time because of the decision making process there will be an increase or decrease in the production of consumption goods. Whether these fluctuations are due to commands from the top in centralized economies or reactions to a volatile market place or just responses to unavoidable delays, machine failures etc is immaterial to the construction of this model. The rate and amplitude of these fluctuations will be of a smaller magnitude than that of the above. We might call this very short term planning. These changes can be divided into two categories depending on how they effect the investment in the economy. A portion of this will have no effect on the investment. Although labor flux is flexible because of the inherent intelligence of human beings, non-human flux is not so and not every type of flux can be used to produce different goods. Something like this could happen if neither the raw materials nor the non-human flux used to produce some consumption goods cannot be used in producing investment goods. The expected contribution to $C(t+\Delta)$ due to these changes will be ;

$$[\sigma(t)\Delta + \vartheta(\Delta)]\Phi_c(t)\, C(t) = \sigma(t)\Phi_c(t)C(t)\Delta + \vartheta(\Delta) \quad (2)$$

where Φ_c is a coefficient of proportionality.

Finally, some of the would-be investment goods -- raw materials and circulating capital--would give rise to (i.e. be "transformed" or "converted" into) consumption goods. Making similar linear assumptions this contribution will be given by

$$[\sigma(t)\Delta + \vartheta(\Delta)]\beta_c(t)I(t)\Delta + \vartheta(\Delta) \quad (3)$$

Thus, adding up the separate contributions we obtain;

$$C(t+\Delta) = [1-\sigma(t)\Delta]\, C(t) + \sigma(t)\,\Phi_c(t)\Delta\, C(t) + \sigma(t)\,\beta_c(t)\, I(t)\,\Delta + \vartheta(\Delta) \quad (4)$$

which can be rewritten as

$$C(t+\Delta) - C(t) = -\Delta\sigma(t)\, [1-\Phi_c(t)]\, C(t) + \Delta\sigma(t)\,\beta_c(t)\, I(t) + \vartheta(\Delta) \quad (5)$$

The derivation above does not purport to give a detailed description of either the decision making processes involved or the workings of the economic system. The derivation is phenomenological. During a time period Δ, which is very small, the total "amount" of consumption goods produced is due to the "autonomous" goods --which we might call planned goods-- and the goods which are an increase or decrease due to very short term changes in those plans. The latter type can either result in an increase/decrease of investment goods or they might be independent of them depending on whether they require the same resources or not. Therefore the constants of proportionality Φ_c and β_c are dependent upon the existing state of technology and also the types of goods being produced.

Dividing Equation (5) by Δ and taking the limit as $\Delta \to 0$ gives;

$$dC(t)/dt = \sigma(t)\, [\Phi_c(t) - 1]\, C(t) + \sigma(t)\,\beta_c(t)\, I(t) \quad (6)$$

A similar analysis for $I(t)$ yields

$$dI(t)/dt = \sigma(t)\, [\Phi_i(t) - 1]\, I(t) + \sigma(t)\,\beta_i(t)\, C(t) \quad (7)$$

Since the coefficients σ, Φ and β have been used only to derive the functional form of the equations , the equations can be rewritten more simply to show the fundamental characteristics of the model. In addition, the non-linearities of the process have been ignored. Thus a more general form of the equations will be;

$$dC(t)/dt = \Psi_{11}(t)\, C(t) + \Psi_{12}(t)\, I(t) + \xi_1(I,C,t) \quad (8.a)$$
$$dI(t)/dt = \Psi_{21}(t)\, C(t) + \Psi_{22}(t)\, I(t) + \xi_2(I,C,t) \quad (8.b)$$

That the "self-coefficients" (i.e. Ψ_{11} and Ψ_{22}) may be positive or negative is clear from the derivation. Also the equations above do not take into account other extensive variables such as capital and labor inputs into the economy.

RELATIONSHIP TO OTHER THEORIES:

The set of equations above have been developed under very general considerations and are not restricted to a single type of an economic system. Hence they relate to other theories developed by other authors. In this section, the relationship of the Equations (8.a) and (8.b) to others will be shown. Denoting the total final product of the economy by $Y(t)$ gives the relationship;

$$Y(t) = C(t) + I(t) \quad (9)$$

The final product can always be expressed as a proportion of the total product

$$Z(t) = \varepsilon(t)Y(t) \quad (10)$$

Since the final product is divided into only two terms, either the investment or the consumption can always be written as a linear function as below;

$$I(t) = \rho(t)Y(t) \quad (11)$$

In either case, the consumption becomes the Keynes-Hicks consumption function. Furthermore, dropping the non-linear terms in Equation (8.a) and substituting for $I(t)$ from Equation (11) results in ;

$$\{d/dt + r(t)\}\, C(t) = \gamma(t)\, Y(t) \quad (12)$$

where

$$\gamma(t) = \psi_{12}(t)\rho(t) \quad \text{and} \quad r(t) = -\psi_{11}(t)$$

Equation (12) has the formal solution;

$$C(t) = \int e^{-\int r(s)\, ds}\, \gamma(t')\, Y(t')\, dt' \quad (13)$$

Equation (13) is essentially the Friedman Consumption Function [2]. Furthermore, adding Equations (8.a) and (8.b) --without the non-linear terms--- results in

$$d/dt\, \{I(t) + C(t)\} = \{\psi_{11}(t) + \psi_{21}(t)\}\, C(t) + \{\psi_{12}(t) + \psi_{22}(t)\}\, I(t) \quad (14)$$

In the event that the equality

$$\psi_{11}(t) + \psi_{21}(t) = \psi_{12}(t) + \psi_{22}(t) \quad (15)$$

is valid, using the definition in Equation (9), we see that Equation (14) can be written as

$$\{d/dt - g(t)\}\, Y(t) = 0 \quad (16)$$

We note that since the coefficients $\psi_{ij}(t)$ are independent, the equality in Equation (15) can hold under very restrictive conditions.

Equation (16) has the formal solution

$$Y(t) = Y(0)\, \exp\{\int g(s)\, ds\} \quad (17)$$

A special case of this solution (i.e. for a constant g) has been derived and passes in the literature as the Harrod-Domar growth model[3]. The Harrod-Domar growth model was derived originally under very restrictive assumptions and suffers from what was called the "the problem of knife-edge growth". Under the derivation shown here, this kind of growth is indeed shown to be very restrictive however the set of Equations (8.a) and (8.b) do not have these restrictions and are thus much more general.

Another specialization of the equations occurs for $\psi_{11}(t) = \psi_{21}(t) = 0$.
This gives rise to;

$$\frac{d}{dt} C(t) = \psi_{12}(t)\, I(t) \qquad (18.a)$$
$$\frac{d}{dt} I(t) = \psi_{22}(t)\, I(t) \qquad (18.b)$$

This set of equations is the two-sector growth model due to Feldman (4).

SUMMARY

An intrinsically probabilistic method has been used to derive relationships between the consumption and investment in a "generic" economic system. Only the aspects or properties of economic systems which will be applicable to any type of economic system have been abstracted and modeled. The result is a set of coupled differential equations which shows only the relationship of consumption to investment. Relationships between the primary inputs --Capital and Labor-- have been modeled in another paper in this Proceedings.

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FOUNDATION FOR AN INTEGER-BASED COSMOLOGICAL MODEL

PART 3 - INTEGERS AND THE NATURAL CONSTANTS

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ABSTRACT

Evidence is presented confirming that the integer-right-triangle (88, 105, 137) has unique number-theoretic properties, thereby further justifying it as the foundational triangle for an integer-based cosmological model. A decomposition of two natural constants reveals that their associated elementary particles are related to each other and to this special IRT.

INTRODUCTION

It has long been speculated that α^{-1}, the reciprocal fine-structure constant, experimentally determined to be about 137.036, should be an integer. In 1932, Eddington wrote: "For my own part I think that its value is exactly 137, that being the number of degrees of freedom associated with the wave-function for a pair of charges" [1]. However, there has been no deductive justification for using exactly 137, and researchers have been confined to the non-integral recommended value of 137.03604(11), as given in the CRC Handbook of Chemistry and Physics (68th Edition, 1987-88 p. F186), where the (11) indicates 1-standard-deviation of uncertainty in the last digits.

The discovery reported in [2], that the orthogonal vector sum of 137 and π is 137.036015719877575, suggests that this might be the true value of α^{-1}. It agrees with the recommended value to at least 7 significant digits, and the presence of π causes no conceptual problems.

Since α^{-1} is the ratio of the velocity of light to that of an electron in orbit, the vector sum suggests a photon travelling cycloidically around a forward velocity component of 137 units and rotating orthogonally at ½ a cycle per unit, with its 'surface' always in contact with the line of forward motion, thereby demonstrating its "spin".

The only IRT having a hypotenuse of 137 is (88, 105, 137) and the relationship between this IRT and other physical constants re-inforces the hypothesis that $\alpha^{-1} = \sqrt{(137^2 + \pi^2)}$ is more than coincidental:

(1) The mass ratio of muon to electron, m_μ/m_e, is recommended as 206.76865(47). It is readily verified that

$$206.768941240526 72 = \sqrt{(207^2 - (88/9)^2)} = \sqrt{(12^2(105-88)^2 - (88/9)^2 + 9 \times 137)}$$

where 88, 105 and 137 are directly from the IRT (88, 105, 137), and 9 and 12 are respectively K and p/K, both of which are discussed below.

N.B. The critical nature of the terms is demonstrated by adding just 0.5 under the radical giving $\sqrt{(207^2-(88/9)^2+0.5)} = 206.77015$, which is beyond the recommended upper limit of 206.76912, using 1-standard-deviation of uncertainty.

(2) The mass ratio of proton to electron, m_p/m_e, is recommended as 1836.15152(70). It is readily verified that

$$1836.15126531988168 = \sqrt{(1836^2 + (88/9)^2 + (193/9)^2)} = \sqrt{(9^2 12^2(105-88)^2 + (88/9)^2 + ((105+88)/9)^2)}$$

where it is observed that the term $(88/9)^2$ also occurs in the m_μ/m_e case but with opposite sign, and so is cancelled when obtaining the sum of