

Paint with numbers

Monte Carlo simulation to determine ratio of water to land on Earth.

For this activity, students simulate randomness as they try to determine the ratio of water to landmass on Earth (unbeknownst to them). Discussions center on the use of random behaviors to estimate characteristics of certain phenomena.

I. About the lesson

1. Mathematics content and process learning objectives
 - a. Experimental vs. theoretical probability
 - b. Ratios
 - c. Percentages
 - d. Asking questions
 - e. Conjecturing
 - f. Error analysis
2. Related creativity traits
 - a. Making connections
 - b. Questioning norms
 - c. Being inquisitive
3. Other disciplinary connections including to everyday life
 - a. When can monte carlo simulations be used to estimate a phenomenon?
 - b. When are random simulations the only practical way to determine a phenomenon/characteristic?

II. Preparing for the lesson

1. Materials
 - a. At least 8 poster paper, prepared as below
 - b. Paint (that is easy to wipe off)—finger paint works well
 - c. Paint brushes—one for each team
 - d. Cups
 - e. Tape
2. Preparation
 - a. Project a map of the world onto the wall so that it can be covered with 8 sheets of poster paper. Poster paper with a “sticky” side works well (like giant post-it notes).
 - b. Draw the outline of each continent (and major islands) on the poster paper, shading in the landmasses
 - c. In the end, you will have 8 poster paper, with each representing 1/8 of Earth. Be sure to label each on the back so that you can re-order the poster paper in the same way
 - d. Make eight cups of diluted paint, about $\frac{1}{2}$ cup each

III. Conducting the lesson

1. Setting up for the exercise.
 - a. Make eight groups. (You can adjust the number of poster papers to use based on the number of groups you'd like.)
 - b. Each group receives
 - i. One poster paper, prepared as above
 - ii. One paper cup with diluted paint. We recommend a dark paint.
 - iii. One paint brush
 - iv. A recording sheet
 - c. Students go outside and take turns and use the paintbrush to splatter dots of paint onto their poster paper. *At this point, they do not know what is represented on their poster paper, nor do they know why they are being asked to do this.*

2. Give the following instructions to perform the experiment:
 - a. Find a spot outside
 - b. Put your poster paper on the ground. Make sure it lays flat
 - c. Each person takes turns
 - i. Dip your paintbrush in the cup of paint
 - ii. Lightly splash the poster paper from a standing height. We want the paint to fall in a scattered way onto the poster paper. No blotches—we want identifiable dots of paint.
 - iii. Stand back as others do the same
 - b. Continue the process until everyone on your team has splashed and the paper has about at least 90 dots. This may mean everyone goes once or twice.
 - c. Fill in the following table:

	Number	Percentage of total dots
Dots in shaded regions		
Dots in unshaded, white regions		
Total number of dots		

3. Analyzing the data. Back in the classroom put the 8 poster papers together on a wall. At this point, students should be able to recognize it as the map of the world. Ask students the following questions:
 - a. What is the significance of the activity?
 - b. What can we determine from each group's table?
 - c. Use the data from the whole class to draw some conclusions?
 - d. Do you think the results of your collective analysis are an accurate estimate of the percentage of Earth that is water, for example?
 - e. Why is it accurate? Why is it not accurate?
 - f. What would have made the estimate more accurate?

- g. What is the effect of the panel that is mostly the Pacific Ocean on the data?
- h. How does the fact that we've "flattened" Earth affect our estimate?

4. Making connections

- a. When might gathering data through randomness produce a "good enough" estimate of real-life phenomenon? (e.g., census, marketing)
- b. Have you used this technique in your other classes? At your job? In your everyday lives? (e.g., estimating number of cells, scheduling shifts at work according)

IV. Assessment

1. The discussion questions could be given as a homework assignment.
2. You could have students design a similar experiment for a different phenomenon (area of the local park, or number of student who are full-time vs. part-time) using monte carlo simulations.

V. Modifications to this lesson

1. In a lesson that follows, we describe making connections between this and a similar activity where students use a globe to estimate the same ratio using monte carlo simulations.
2. You can also derive/confirm the area of various figures (e.g., circles) using this technique