

Euclidean and non-Euclidean geometry.

For this activity, students work on problems to test their understanding of the axiomatic foundations of geometry. By challenging their assumptions about geometry, the exercise aims to help them discover the fundamental laws of non-Euclidean geometry and the significance of context to underlying mathematical laws. This module can be implemented over three hour-long class periods.

I. About the lesson

1. Mathematics content and process learning objectives
 - a. Euclid's axioms of geometry.
 - b. Euclidean Distance.
 - c. Pythagoras theorem.
 - d. Non-Euclidean geometry.
 - i. Spherical Geometry.
 - ii. Hyperbolic Geometry.
 - iii. Projective Geometry.
 - e. Difference between Euclidean and non-Euclidean Geometry.
2. Related creativity traits
 - a. Questioning norms
 - b. Being inquisitive
 - c. Making connections.
3. Other disciplinary connections including to everyday life
 - a. Using Non-Euclidean geometry and measuring distances on the globe.
 - b. Using Projective geometry in art.

II. Preparing for the lesson

1. Materials
 - a. Strings.
 - b. Protractors
 - c. Yard sticks
 - d. Rulers
 - e. Graph paper
 - f. Printout of globe.
 - g. Worksheets containing some basic problems involving Euclidean geometry such as distance formula, rules about triangles and Pythagoras theorem.
 - h. Plain beach-balls.
 - i. Marker pens.
2. Preparation
 - a. Print out map of the Earth showing the following 3 cities: Buffalo, NY; Quito, Ecuador and Nanyuki, Kenya.

- b. Project a map of the world (google map) onto a screen showing these 3 cities.
- c. Find actual distances from Buffalo to Quito & Quito to Nanyuki.
- d. On the beach-balls draw out a few triangles of different sizes.

III. Conducting the lesson

Exercise 1: Invalidity of Euclidean Distance formula on curved surfaces.

1. Setting up for the exercise.
 - a. Students work individually on a worksheet to refresh their understanding of the
 - i. Euclidean distance formula.
 - ii. Rules about Euclidean triangles.
 - iii. Pythagorean theorem.
 - b. Each group receives the printout of the map of the Earth containing the actual distances from Buffalo to Quito & Quito to Nanyuki.
 - c. Students in each group were asked to compute the distance from Buffalo-Nanyuki.
2. Give the following instructions to perform the exercise:
 - a. Using the distance formula/Pythagoras theorem, compute the distance from Buffalo-Nanyuki and compare with the known value of this distance.
 - b. Compute the % error.
3. Class Discussion:
 - a. In groups of 3-4, discuss the possible reasons for this error.
 - b. Record important keywords on the blackboard.
 - c. Through the discussion, get students to consider the importance of curvature on this apparent discrepancy in their calculations.

Exercise 2: Triangles on Curved Surfaces.

4. Setting up for the exercise.
 - a. Put students in groups of 3-4.
 - b. Students are given a preparatory worksheet which gets them thinking about Euclidean triangles. The worksheet contains a few exercises asking them to measure interior angles of several triangles and calculate their total.
5. Give the following instructions to perform the exercise:
 - a. Each student is then given a beach ball containing several triangles drawn on its surface.

- b. Students are asked to measure the interior angles and the sum of these angles for different triangles on this surface.

6. Classroom Discussion.

- a. Students are asked to state the totals that they obtained from their measurements.
- b. These values are recorded on the blackboard.
- c. Students are asked to discuss their observations. Specifically, they will be asked to think of the following questions:
 - i. What do they notice between Euclidean and non-Euclidean triangles?
 - ii. What might be the cause of this difference?
 - iii. How much can non-Euclidean triangles vary?
 - iv. On the sphere, what the largest value of the total angle?
 - v. What are interior and exterior angles in a triangle?
 - vi. How does the concept of interior and exterior angle change for triangles on a sphere versus those on a flat sheet of paper?
 - vii. How are open and closed surfaces different?

7. Making connections

- a. How might the results of the exercises be pertinent in our daily lives?
- b. For instance, how could the adjustments to our understanding of geometry on curved surfaces be pertinent in the context of long distance traveling on the Earth?

IV. Assessment

1. The discussion questions could be given as a homework assignment
2. As part of the homework assignment, students could be asked to
 - a. Find other types of curved surfaces (such as a saddle shape geometry of a “Pringle potato chip”)
 - b. Repeat similar calculations to those done in class on these surfaces and consider how their results change.
 - c. Try to identify larger patterns.
3. Along the same lines, students can be asked to think about the meaning of parallel lines on a flat surface and then shown art that shows parallel lines meeting “eventually”. They can be asked to think about how our concept of parallelism changes on a spherical surface.

V. Modifications to this lesson

1. This lesson can be conducted on different surfaces in classroom – such as a saddle.
2. Questions discussed as part of the Assessment section can be discussed in the classroom depending on how much time is available to discuss these ideas in the class.

3. A potentially interesting way to motivate this discussion is by using examples from Astronomy. The gravitational lensing effect, whereby one can see celestial objects hidden behind another massive celestial body, is caused due to the bending of the light in space. The importance of Non-Euclidean geometry in space travel could be discussed in class.