Euclidean Geometry

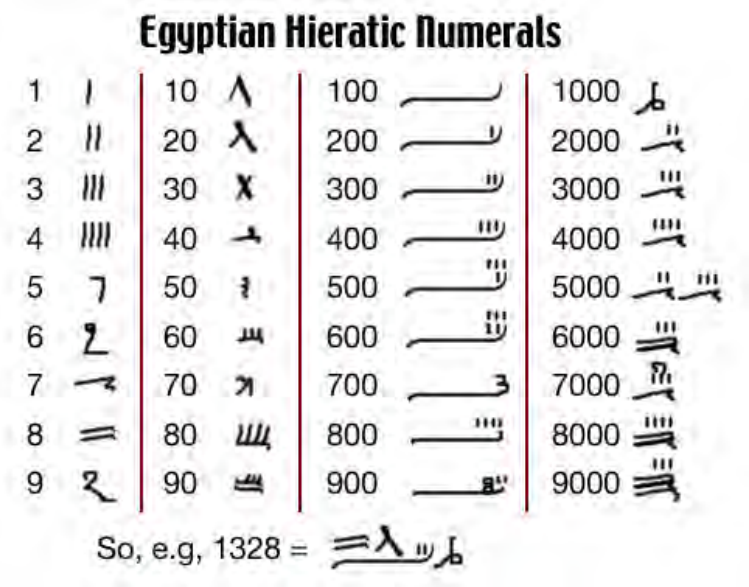
January 24-26, 2017

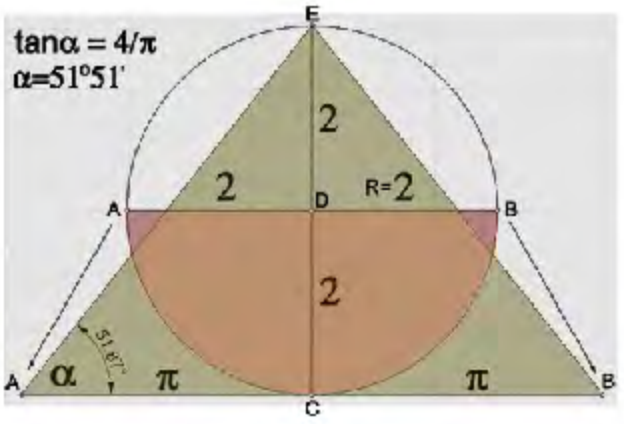
Geometry

- Geometry = geo(earth) + meter (measure)
- The first ideas of geometry come from nature:
  - the shapes of mountains resemble triangles
  - The sun and moon are circular in the sky
  - Flowers and fruits display more complex shapes

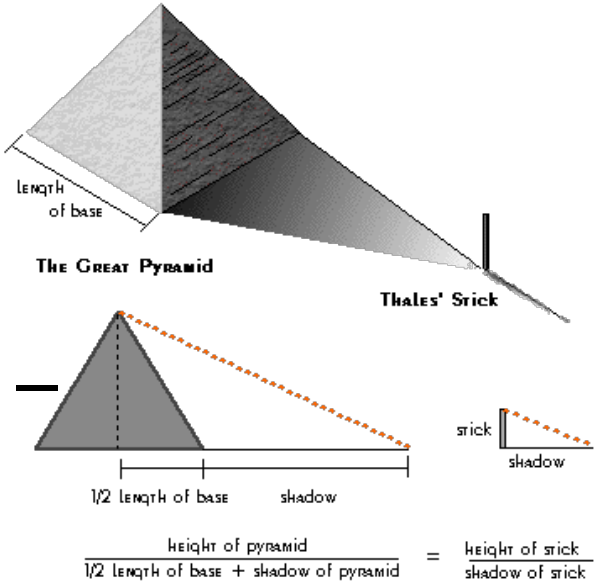
Historical development

- Egyptian civilization
  - Pyramids

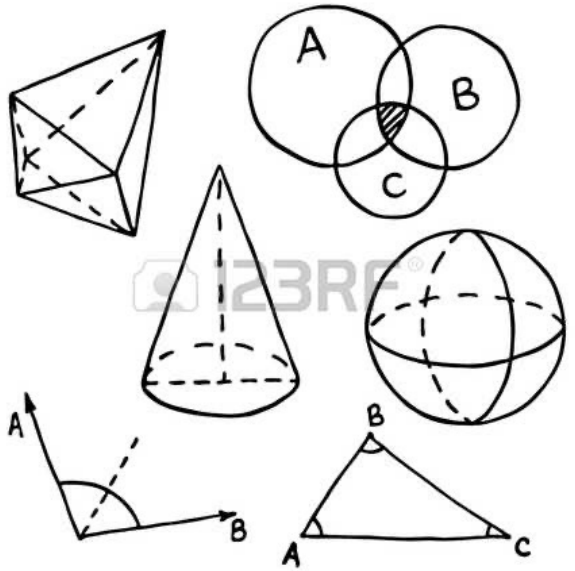




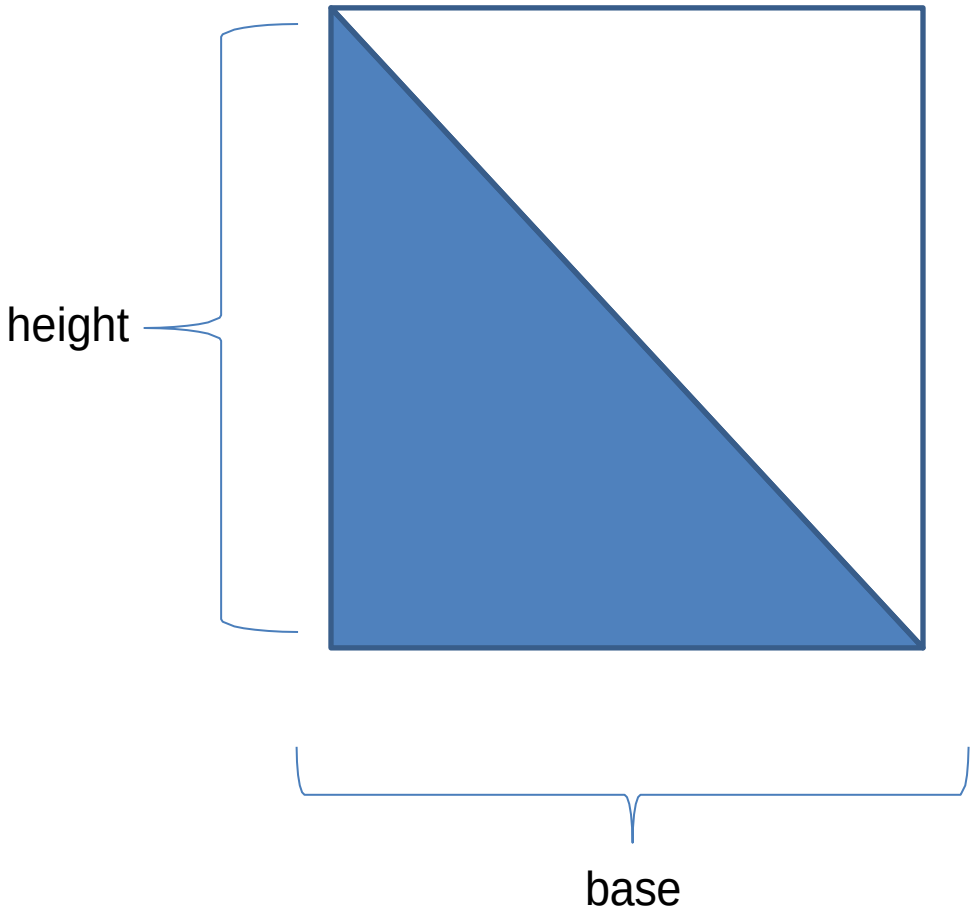
- Greek civilization
  - Thales (624-548 BC)



  - Pythagoras (580-500 BC)

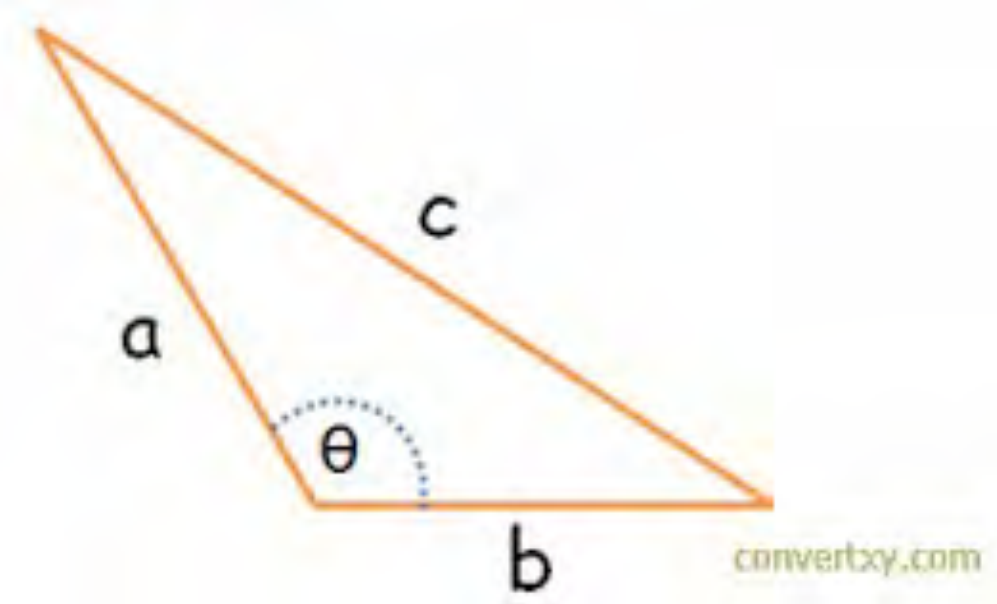


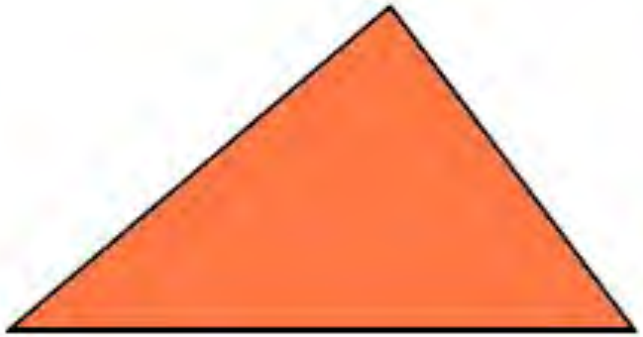
The area of a triangle



Area of triangle = ½ base x height

Class Exercise 1

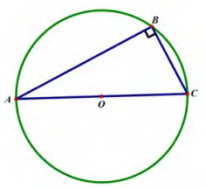




- Find the area of these scalene triangles.

The Theorems of Thales

1. A circle is bisected by a diameter.
2. The base angles of an isosceles triangle are the same.
3. The pair of vertical angles formed by the intersection of two lines are equal.
4. Two triangles are **congruent** if they have two angles and included side equal.
5. An angle inscribed in a semi-circle is a right angle.



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Conditions for Congruence of Two Triangles

SSS *(Side – Side – Side)*

3 sides are respectively equal

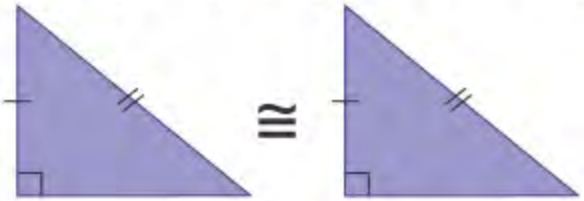
SAS *(Side – Angle – Side)*

2 sides and the included angle are respectively equal

ASA *(Angle – Side – Angle)*

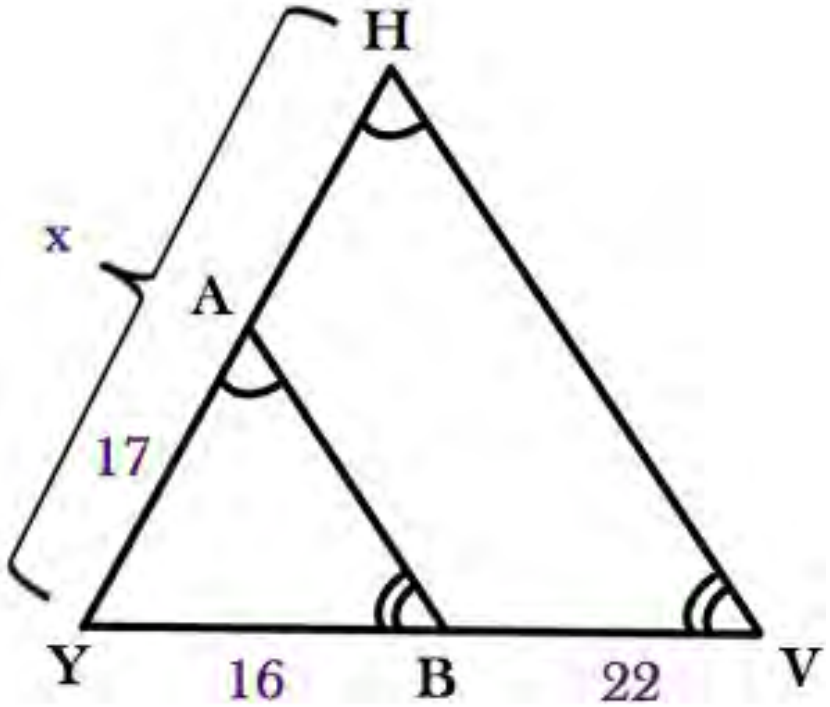
2 angles and the included side are respectively equal

RHS *(Right angle – Hypotenuse – Side)*



Hypotenuse and one side are respectively equal

Similar triangles



- Triangles AYB and HYV are **similar.**
- These triangles have 3 angles the same (AAA) but not the sides.

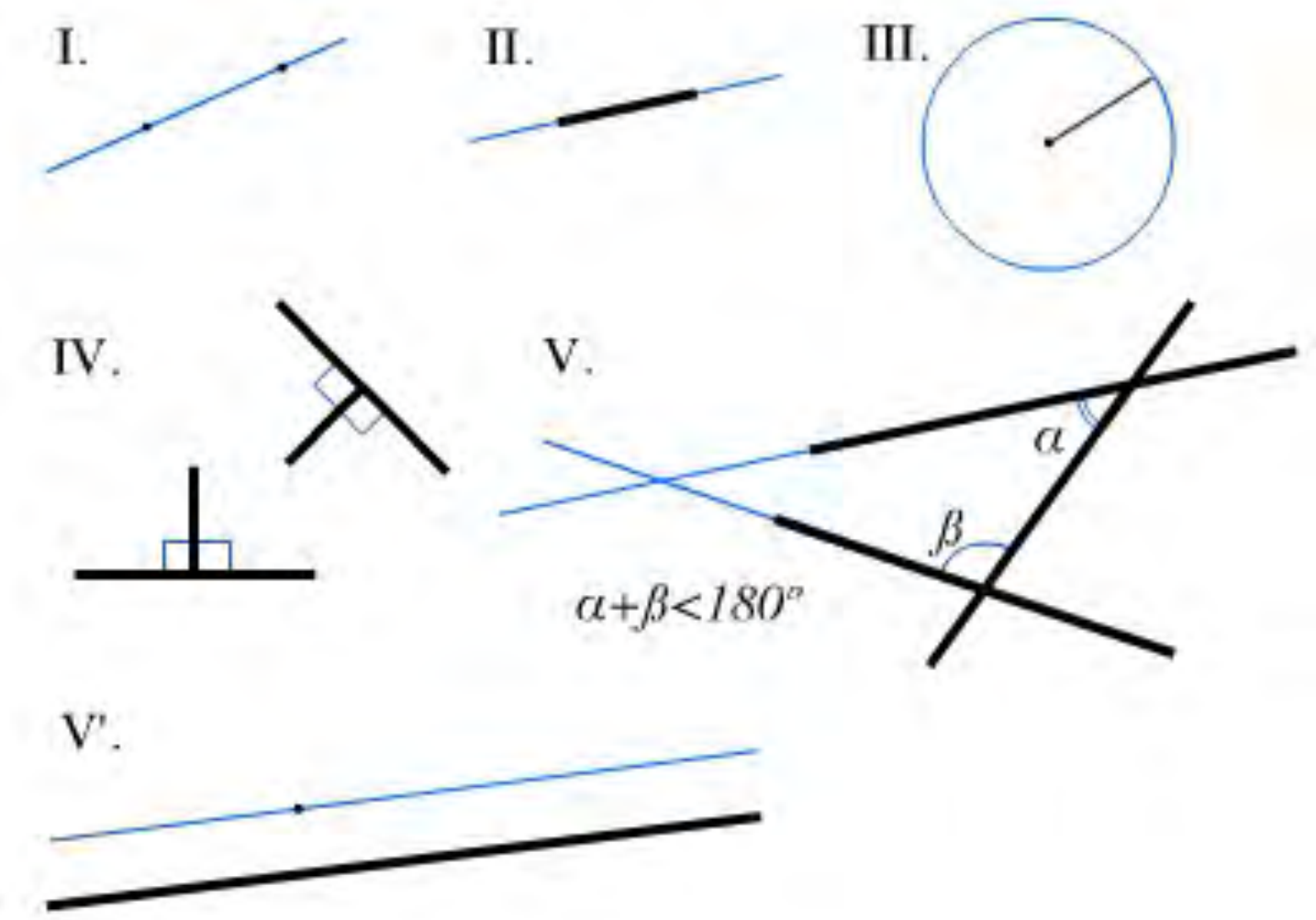
Axiomatic Method

- "The term **axiomatic geometry** can be applied to any **geometry** that is developed from an **axiom** system, but is often used to mean Euclidean **geometry** studied from this point of view." (Wikipedia)

Euclid's postulates

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given a straight line segment, a circle can be drawn having the segment as a radius and one endpoint as center.
4. All right angles are congruent.
5. Given any straight line and a point not on it, there exists one and only one straight line which passes through that point and never intersects the first line.

Euclid’s postulates

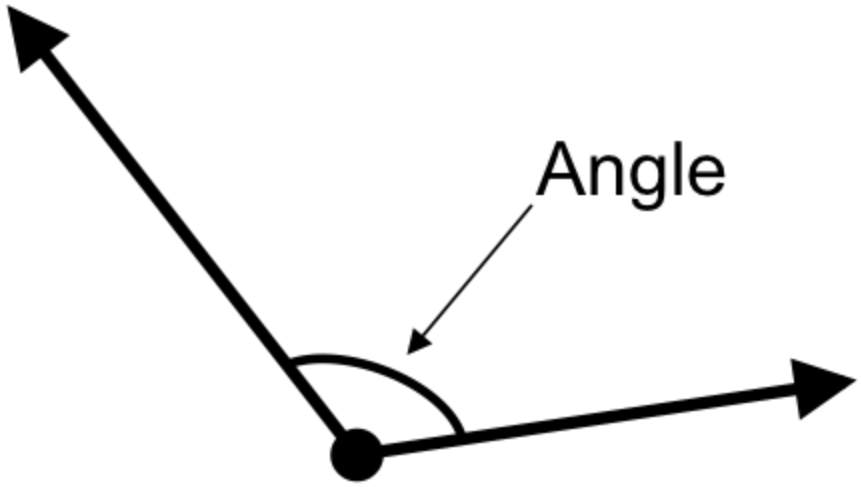


Class exercise 2

- Consider the following axioms:
  1. There are 3 students.
  2. For every pair of students, there is exactly one class in which they are enrolled.
  3. Not all students belong to the same class.
  4. 2 separate classes share at least one student in common.

What can you deduce from these axioms?

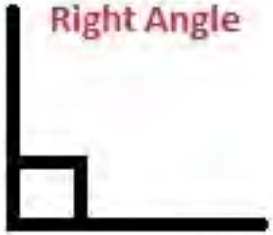
Definition of angles



Acute Angle

Less than 90°

Right Angle



Exactly 90°

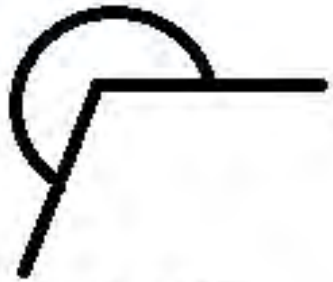
Obtuse Angle

Greather than 90° but less than 180°

Straight Angle

Exactly 180°

Reflex Angle

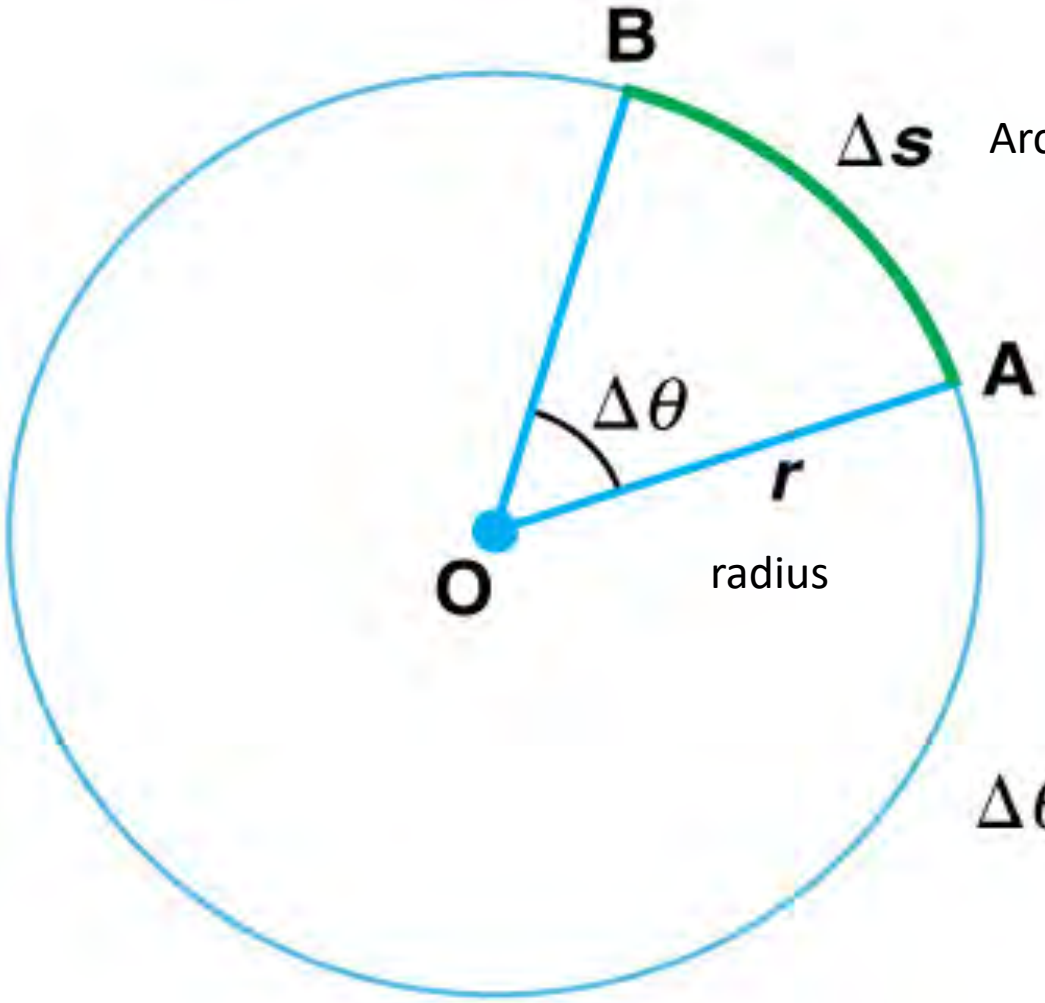


Greater than 180°

Full Rotation

Exactly 360°

Definition

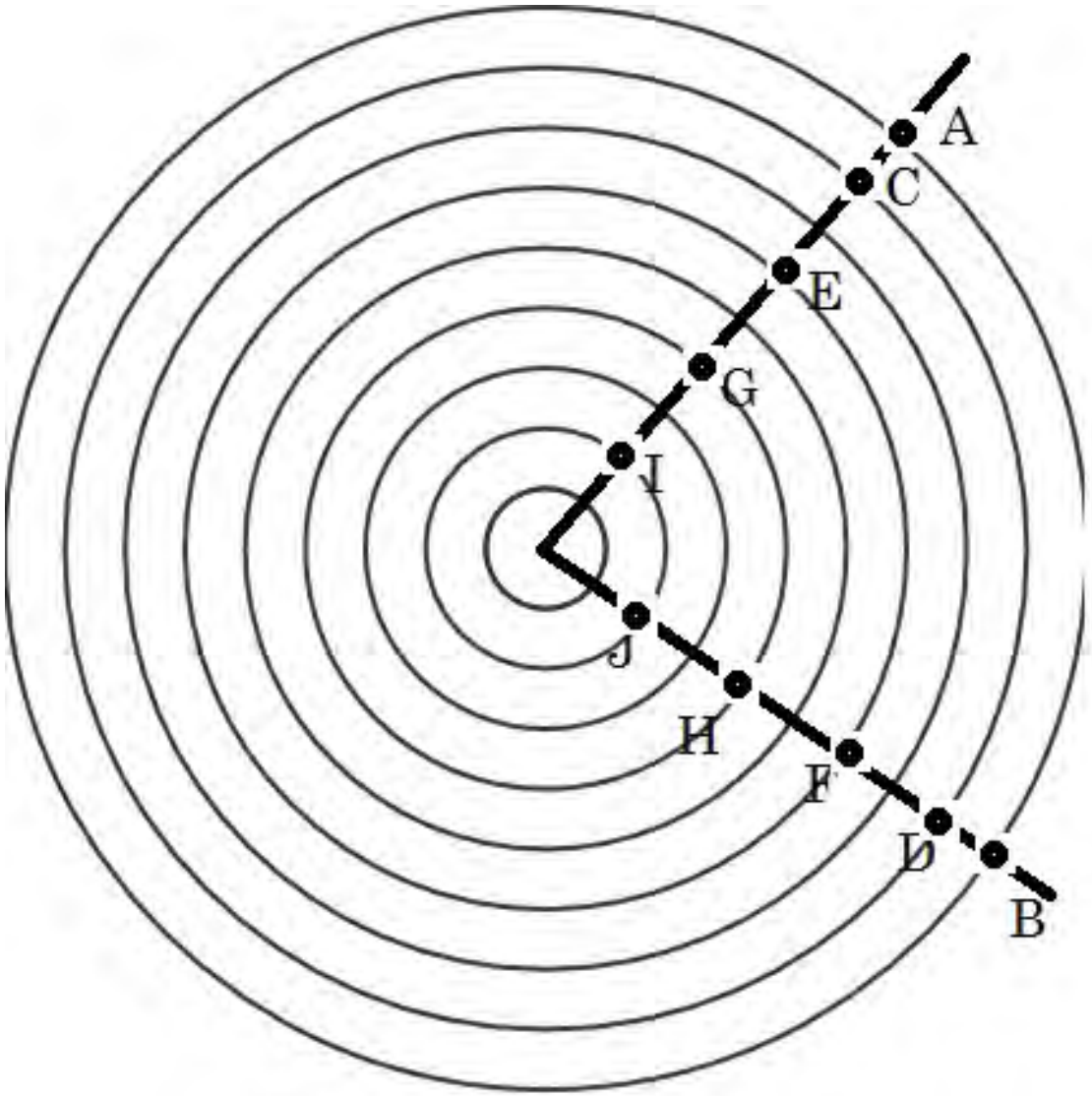


$$\Delta\theta = \frac{\Delta s}{r}$$

How we measure angles

- The radian is one measure of angle. In radians, the angles can vary between 0 and 2π.
- What is the circumference of a circle of radius r? Can you relate this to the definition of the angle provided earlier?
- Angles can also be expressed in degrees, between 0 and 360.
- How is a radian related to a degree?

Class exercise 3
Consider the following picture

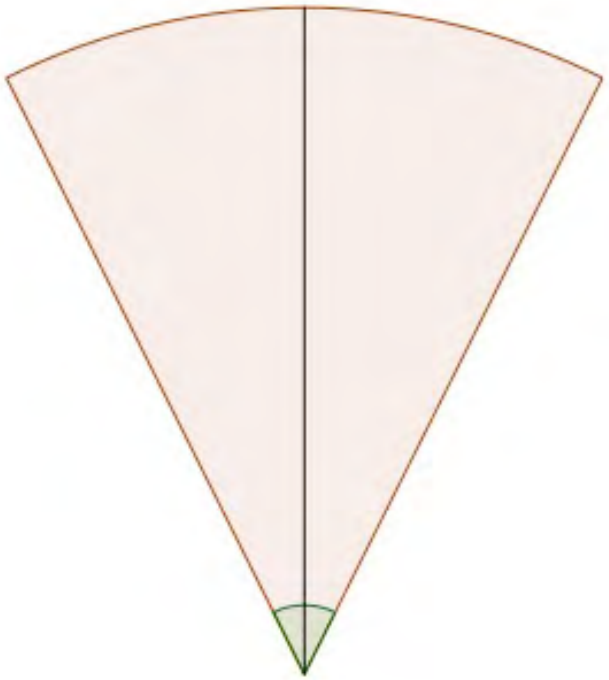


Exercise 2 - contd

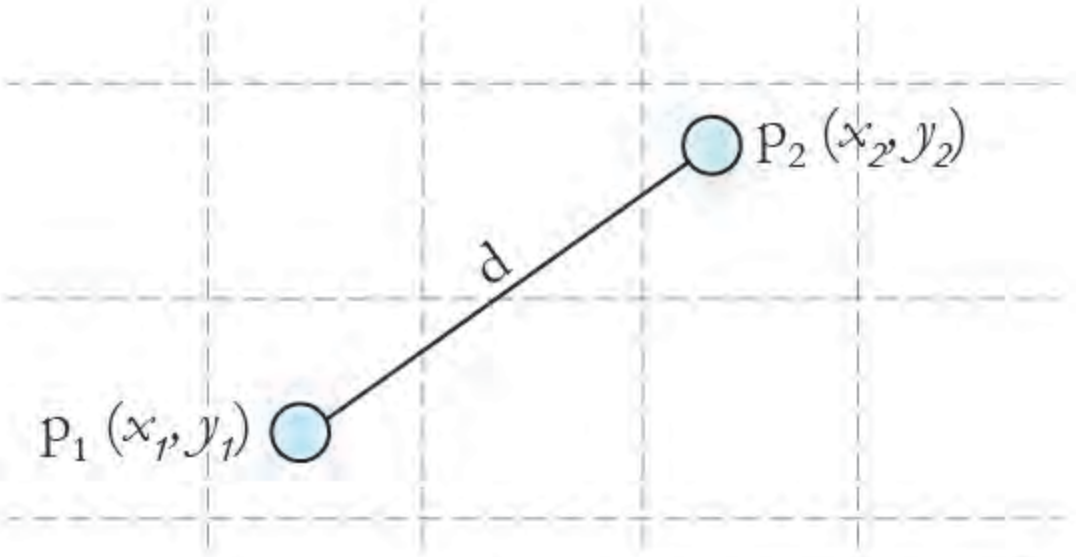
- Find the arc length, radius and angle corresponding to segments:
  - AB
  - CD
  - EF
  - GH
  - IJ
- What underlying pattern do you think exists in this picture?
- Graph the arc length vs. radius for the different segments.
- What do you infer from this?

Class exercise 3

- Arc length lab (ropes and meter sticks)

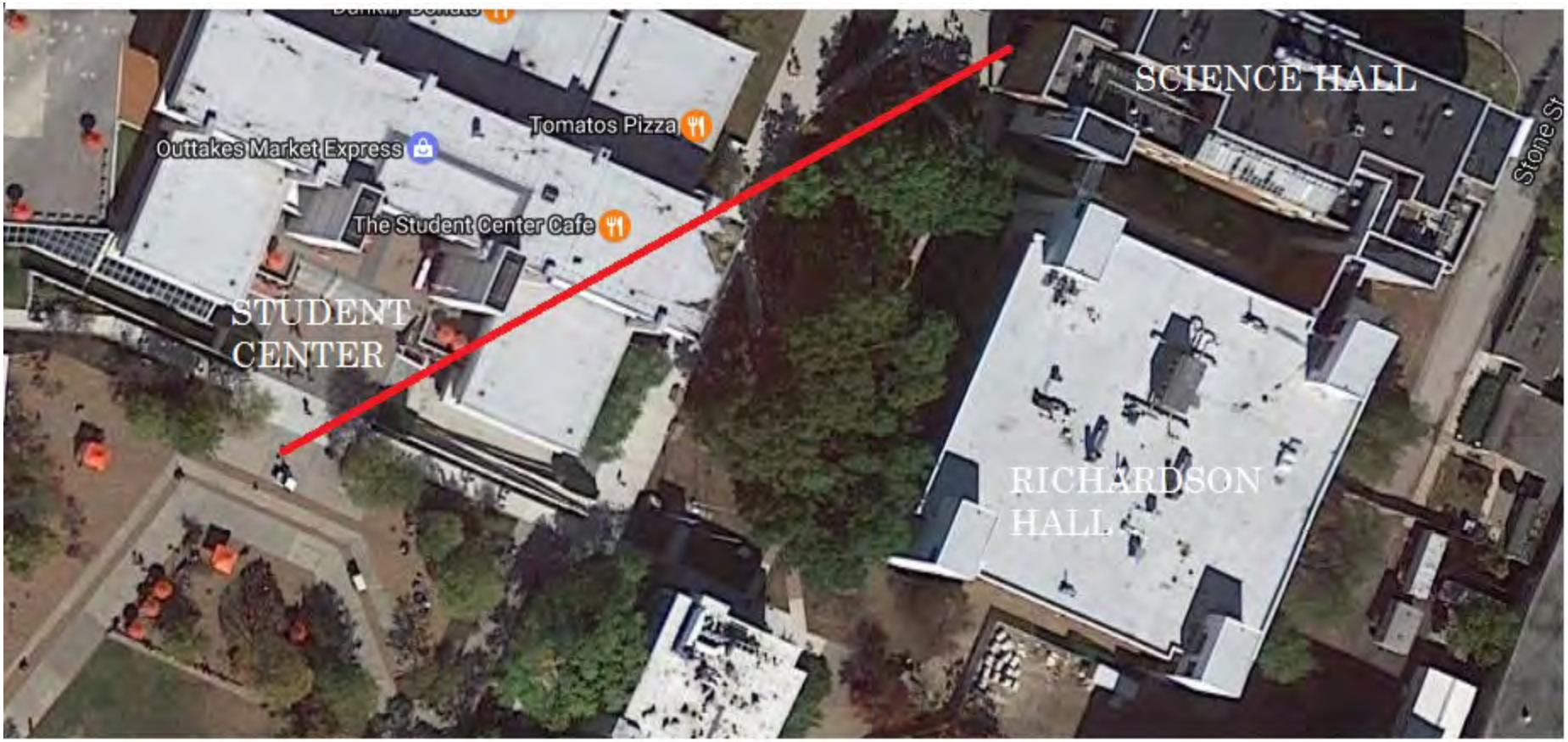


Euclidean distance



Euclidean distance (d) $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Class exercise 4



- Find the distance between the front of Science Hall and the spot in front of the Student center, as indicated by the red line in the picture.

Homework problem 1

- Consider the following 3 axioms:
  - *There are 5 flavors of ice cream: Vanilla, Chocolate, Strawberry, Bubble Gum and Cookie Dough.*
  - *Given any 2 flavors, there is one child who likes these 2 flavors.*
  - *Every child likes exactly two different flavors among the five.*
- Use these axioms to determine how many kids there are. Show the logic behind your work and also show that your result satisfies each of the axioms.

Homework problem 2

- Use the concepts discussed in class to determine the height of the highest point of Richardson Hall.
- The only tools you are allowed to use are a pencil and a 12inch ruler.

- Explain your thought process and show all calculations. Include all the ideas that you thought of, even if you did not try them out.