The Development of Failure to Learn Mathematics (FtLM): 
a discursive perspective

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Failure to Learn Mathematics (FtLM) – huge amount of research, and yet still many gaps

Cognitive perspective: Explains FtLM as a problem in the development of a certain mental capacity
(Butterworth, 2005; Geary, 2011; Dowker, 2019)

• Mostly relates to FtLM as a state
• Main terms: Dyscalculia; Mathematical Learning Disability; Mathematical competence

Affective perspective: Explains FtLM (partially) in terms of “math anxiety” or “low math concept”, or “low motivation”
(Dowker et al., 2016; Ma, 1999; Ramirez et al., 2018)

• Relates to FtLM as a process (reciprocal relations between affect and achievements), but at a very macro-level
• Main terms: Math anxiety; math self-concept; motivation to learn mathematics; self-efficacy in mathematics

Social perspective: Explains FtLM in terms of marginalization, lack of opportunities to learn, and social narratives inhibiting engagement in learning
(Ben-Yehuda et al., 2005; Martin, 2000; Tan et al., 2019)

• Relates to FtLM mostly as a state, sometimes as process, rarely relates to the mathematical content
• Main terms: race; gender; home-environment; SES
Cognitive explanation: FtLM happens because of inherent/developmental deficits

- But then: why is math anxiety so strongly correlated with achievements (and the relation is bi-directional)?
- Why are there more MLD in low SES?
- Why is math achievement so strongly correlated with SES? With Gender (in certain countries)?

Affective explanation: FtLM happens b/c students develop apprehension towards math

- But then – why do certain cultures have high incidents of anxiety and high achievements?
- Where does the anxiety come from? How does it develop?
- How does the anxiety affect learning (not just test taking)?

Social explanation: FtLM happens b/c students of certain identities are not given sufficient OTLs

- But then – why do certain students with similar identities fail and other succeed?
- How does the identity label impact OTLs?
There is need for a holistic perspective
Commognition (Sfard, 2008) in a nutshell

Major underlying assumptions
- Thinking is an intra-personal form of communication
- Mathematics is a discourse
  - Defined by certain keywords, visual mediators, narratives and routines
- Learning is becoming a participant in a community

The theory
- Learners start by participating ritually – imitating others
- Gradually, their participation becomes more explorative, as they master the routines of the discourse

If we want to study learning, let us study students’ discourse
Identity in communicational terms

“A collection of stories, told about a person, that are reifying, significant and endorsable”
(Sfard & Prusak, 2005)

Types of identity stories:
• 1st Person, 2nd P, 3rd P stories
• Current identity: stories about the current situation
• Designated identity: stories about how things should be
From Identity to Identifying (product to process)

Mathematizing

- Communicating about mathematical objects

Subjectifying

- Communicating about the participants in the discourse

Identifying

communicating about what people are or have
Levels of subjectifying

Specific
"I don’t think I solved this right"

General
"I always fail on this in tests"

Properties/Membership
"I’m not good with fractions"

Implicit identifying: Actions intended to elicit in others identity narratives about oneself
Failure to Learn Mathematics - A discursive definition

A consistent impasse to developing an identity of inclusion in a certain mathematical community/discourse
Learning as progressing from rituals to explorations

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<tr>
<th>Ritual Routines</th>
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Discourse on algebra (meta-arithmetic)
Discourse on rationale numbers
Discourse on natural numbers
Learning as progressing from rituals to explorations

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FtLM is the process whereby students get “stuck” in the ritual phase of learning

Lavie, I., Steiner, A., & Sfard, A. (2019); Sfard & Lavie (2005)
The story of Idit

- 7th Grader (Age: 13)
- Moderate-high achievements
- Identifies herself as generally excelling in math
- Her only problem is that she’s “not good with fractions”
- Participates in my out-of-school course, succeeds at final year exams, gets tracked to the highest track

- End of 9th grade: Idit fails math; gets tracked to the lowest (basic) high-school math track
- Reports severe “math anxiety”

Idit’s 1st math interview – 7th grade

Idit: So Um.. (looks at interviewer, smiles) here I’m completely unsure, I told you I’m not good with fractions. (Looks back at the sheet), so I think… that this is bigger (points to 1/5).. [I’m] not sure.

Interv: Why do you think so?

Idit: ‘cause, if I’m not mistaken, in elementary school we were told that the smaller the number, so it’s bigger. That’s what we were told (shrugs shoulders, slightly smiles).
Mathematizing

The task: write >,< or =:  \[ \frac{1}{5} \] \[ \frac{1}{7} \]

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Analysis of ritual/explorative participation

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Syntactic mediation
1 / 5 / 7 treated a separate signs

Reified identity narratives of failure (specific to fraction)

Lack of agency

Reliance on external authority
Idit's discourse on natural numbers

Idit: (Reads the question) Um.. Four shirts cost 200 $, six pairs of pants cost 300$. Rina bought two shirts and two pairs of pants. How much did she pay? So she bought two shirts which is 100 $ (looks down)

Interv: Umhmm

Idit: And two pants which is... also 100 $

Interv: OK

Idit: ( Writes) so that gives 200$
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Interv: Umhmm

Idit: And two pants which is... also 100 $

Interv: OK

Idit: (Writes) so that gives 200$
Idit’s identity narratives at 9th grade

Idit: “The instant I get to a test it like, gives me a blackout, that I forget everything. Now, before the test, I know all the material, and- and I go over everything, I solve all the exercises in the book and I know everything by heart. And- and when I get to the test I have this, like suddenly this anxiety that makes me forget all the material.”

Mother: “She has an expectation of herself too, (its) not just that she knows that we have an expectation (of her). But we don't come (.) or punish (her). She knows that we just care that her grades reflect what she knows, we know that she knows.”
Idit’s hypothesized widening failure

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<td>Objectified mediation</td>
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<tr>
<td>(meta-arithmetic)</td>
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- Discourse on natural numbers
- Negative feedback from teachers, tests, HW
- Negative identity narratives, still local to fractions
- Disengagement from OTL fractions
- Seeking “shortcuts”
- Ritual participation in specific domain
- Widening of negative identity narratives
Dana’s hypothesized widening failure — probably started much earlier

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**Seeking “shortcuts” + Only ritual OTLs**

**Negative feedback from teachers, tests, HW**

**Ritual participation in specific domain**

**Widening of negative identity narratives**

Discourse on algebra (meta-arithmetic)

Discourse on rationale numbers

Discourse on natural numbers

Discourse on numbers between 1-20

28/04/2022 14:41
Researching ways to assess and intervene with FtLM

• The LATID project: Learning Algebra through Technology for developing mathematical Identities

• The main problem:
  • If students arrive at 7th grade with a ritual arithmetic discourse how can they ever participate exploratively in the meta-arithmetic discourse = algebra?

• The solution:
  • By using Excel activities: Provide students with scaffolds to “bypass” the ritual routines in arithmetics, so they can participate exploratively in the discourse of algebra

Prof. Michal Tabach, Tel-Aviv University

Funded by the Israeli Science Foundation Grant 744/20
The LATID project: Learning Algebra through Technology for developing mathematical Identities

• Four year project (Current: Year 2/4)
• Stages:
  • Develop diagnostic tools to assess arithmetic and early algebraic discourses
    • RDP – aRithmatic Discourse Profile
    • EADP – Early Algebra Discourse Profile
  • Intervene with Excel based activities
  • Measure the effectiveness of B with the tools developed in A, as well as through analysis of learning processes.
RQ: Based on tasks taken from elementary school curriculum, how can students’ arithmetic discourse be characterized on the continuum between ritual and explorative participation?
Method

• 12 7th grade students (Heyd-Metzuyanim, 2011), interviewed on arithmetic tasks using a “think aloud” protocol

• 3 tasks chosen for the present analysis: 96+7935; 25x99; $\frac{2}{3} \times 9$

• Verbatim transcriptions

• 1st Step of analysis: dividing each routine into subroutines

• 2nd Step: determining ritual/explorative characteristics of the routine
## Criteria of analysis for ritual-explorative routines

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Analytical actions</th>
<th>Characteristics of an explorative routine</th>
<th>Characteristics of a ritual routine</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Objectified/syntactic mediation.</td>
<td>Searching for evidence that the nouns signify numbers/quantities and not just the signifier of the number</td>
<td>In whole numbers, relating to the place value of the numeral. In fractions: relating to different realizations of the fraction as the same, including fraction as operator, part of whole, part of quantity, etc.</td>
<td>In whole numbers: relating to operations as signaling procedures on digits rather than on the whole number. In fractions, relating separately to the numerator and denominator.</td>
</tr>
<tr>
<td>2 Flexibility</td>
<td>Look for multiple procedures that are associated with the same task.</td>
<td>More than one procedure is associated with the main task OR a non-standard procedure is applied to the task.</td>
<td>Relying on only one procedure while showing rigidity and reluctance to use any other procedure.</td>
</tr>
<tr>
<td>3 Agency/External authority</td>
<td>Look for subjectifying discourse; examine verbs/pronouns and non-verbal signals that indicate the confidence.</td>
<td>Mathematizing with high confidence (no hesitations, question marks, no looking for approval).</td>
<td>Talking with question marks; Verbally or Non-verbally seeking approval from the interviewer; Relating to external authority for justification (e.g. “that’s what I learned in school”)</td>
</tr>
<tr>
<td>4 Focus on goal or on procedure</td>
<td>Look for verbs indicating doing (e.g., “I add”) vs. being verbs indicating the result (“it is...”);</td>
<td>Talking about the result, checking it, or explaining it spontaneously</td>
<td>Talking about the actions of the procedure. Ending the procedure without relating to the reasonableness of the result.</td>
</tr>
<tr>
<td>5 Bondedness</td>
<td>Examine the procedure; sub-procedures and the bonds between them.</td>
<td>Each sub-procedure feeds the next sub-procedure. The narrative of the result of sub-procedure N serves as the input of sub-procedure N+1.</td>
<td>There is a disconnect between a certain sub-procedure and its following one OR sub-procedures using different realizations are not treated as the same.</td>
</tr>
</tbody>
</table>
## Findings – Ritual-exploration ratios and relative placement of the 12 students

<table>
<thead>
<tr>
<th>Student</th>
<th>Achievement group</th>
<th>96+7935 Ratio Ritual/Exploration</th>
<th>25×99 Ratio Ritual/Exploration</th>
<th>$\frac{2}{3} \times 9$ Ratio Ritual/Exploration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dana</td>
<td>Low</td>
<td>6/4</td>
<td>8/2</td>
<td>7/3</td>
</tr>
<tr>
<td>Hili</td>
<td>Low</td>
<td>7/2</td>
<td>Not attempted</td>
<td>Not attempted</td>
</tr>
<tr>
<td>Hila</td>
<td>Low</td>
<td>7/3</td>
<td>7/1</td>
<td>Not attempted</td>
</tr>
<tr>
<td>Naor</td>
<td>Low</td>
<td>2/6</td>
<td>8/3</td>
<td>4/5</td>
</tr>
<tr>
<td>Edna</td>
<td>Middle-high</td>
<td>8/3</td>
<td>6/7</td>
<td>Not attempted</td>
</tr>
<tr>
<td>Idit</td>
<td>Middle-high</td>
<td>0/7</td>
<td>6/5</td>
<td>7/4</td>
</tr>
<tr>
<td>Dan</td>
<td>Middle-high</td>
<td>1/7</td>
<td>7/6</td>
<td>2/6</td>
</tr>
<tr>
<td>Ziv</td>
<td>Middle-high</td>
<td>3/6</td>
<td>3/8</td>
<td>2/6</td>
</tr>
<tr>
<td>Ram</td>
<td>Excelling</td>
<td>0/6</td>
<td>0/8</td>
<td>1/7</td>
</tr>
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<td>Gabby</td>
<td>Excelling</td>
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**Ritual 8/0**

**Explorative 0/8**
The commognitive explanation of FtLM is theoretically robust, but accumulating empirical evidence is difficult

- Commognitive analysis is very work-intensive
- Demands longitudinal analysis over long periods of time, with very high engagement from participants

Another challenge lies in the *incommensurability* of commognition with other “cognitivist” theories

- Without a “common language”, building on previous studies that have used cognitivist theories is extremely difficult

**Work in progress**: a critical review with the aim of understanding how the highly fragmented literature on FtLM can be built upon and viewed through a discursive lens


