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TESTS OF THE SPECIFICATION OF UNIVARIATE AND BIVARIATE ORDERED PROBIT

J. S. Butler and Patrali Chatterjee*

Abstract—This note presents tests of the specification of univariate and bivariate ordered probit. The test is sensitive to deviations from either normality or the exogeneity of the explanatory variables. As an example, the ownership of dogs and televisions, both sources of time-intensive entertainment, is studied. The specification for dogs is not rejected, the specification for televisions is rejected at the 2.0% level, and the specification of both together is rejected at the 1.3% level.

I. Introduction and Literature Review

Ordered probit models represent situations in which a discrete outcome represents greater affinity, preference, or propensity for a good or outcome. Examples include children or, in this paper, dogs and televisions. The underlying propensity could represent a tendency or quality; examples include discrete quality measures and contract provisions negotiated as a function of bargaining strength.

The assumptions of the ordered probit model include a list of explanatory variables that affect the dependent variable and are exogenous, i.e., uncorrelated with either the normally distributed latent disturbance or the prediction error from the model. Maximum-likelihood estimation (MLE) maintains and efficiently employs the assumptions, but is inconsistent if any of the assumptions are invalid. This note proposes tests of the assumptions of normality and exogeneity using estimation by the generalized method of moments (GMM). The null hypothesis of the GMM test of the specification is a joint hypothesis that the latent dependent variable is distributed normally and that the explanatory variables are exogenous. If the test produces a rejection of the joint null hypothesis and instrumental variables are available, the model could be reestimated by GMM to test separately the effect of normality. If the test does not reject the joint null hypothesis, however, MLE could be used with stronger assurance that specification error is not present.

Ordered probit models are applied rarely in bivariate models. We report here estimation in such a model after testing the model specification. We examine ownership of dogs and televisions, both sources of time-intensive entertainment.

See Maddala (1983) for the earliest uses of ordered probit models in economics. The papers cited here use ordered probit in two-equation models. Jimenez and Kugler (1987), Frazis (1993), and Butler et al. (1994) use ordered probit models as the first stage of a selection bias model. All are studying aspects of education: the effect on earnings of in-service training in a Colombian program, the effect on earnings of schooling choice concerning college, and the effect on grades in intermediate microeconomics of calculus classes. A different two-stage model with ordered probit in the first stage is used by Kao and Wu (1990), who study the default risk of bonds (first stage) and the yield on bonds (second stage). Amel and Liang (1994) model the entry of banks into new markets by probit or ordered probit and, as a second stage, the market performance of banks.

Gustaffson and Stafford (1992) study the decisions of Swedish mothers to work and to receive public child care subsidies. They use ordered probits to model the decision to work in three ranges. Their model does not allow correlated disturbances.

Calhoun (1989, 1991) uses bivariate ordered probit models to study the relationship between desired and excess fertility. The dependent variables are children ever born *CEB* and desired family size *DFS*. *DFS* and *CEB* can be estimated as a bivariate ordered probit, and the *DFS* can be censored in that it can be reported as the number of children ever born, even if the *DFS* is less than *CEB*. The censored model then takes *DFS* as reported if *DFS* exceeds *CEB* or as *CEB* or less if *DFS* is reported to be *CEB*. That avoids asking about unwanted births (*DFS* less than *CEB*). Calhoun (1989, 1991) thus estimates a censored model not used in this paper, but does not test the specification.

II. The Ordered Probit Model and the Bivariate Ordered Probit Model

We begin by specifying the bivariate ordered probit model, then describing the univariate test, and finally describing the bivariate test. We indicate the two ordered probit indexes by subscripts a and b, functions of single indices z_a and z_b, which are functions of exogenous variables $\mathbf{X}$ and coefficients $\boldsymbol{\beta}_a$ and $\boldsymbol{\beta}_b$. The exogenous variables need not be the same in the two equations, and a simultaneous-equations model in the two indexes can be estimated if each equation includes at least one regressor omitted from the other. Subscripts indicating individual observations are suppressed. The unobserved propensities are defined as

$$y_a^* = \mathbf{X}'\boldsymbol{\beta}_a + \epsilon_a = z_a + \epsilon_a \tag{1}$$

$$y_b^* = \mathbf{X}'\boldsymbol{\beta}_b + \epsilon_b = z_b + \epsilon_b. \tag{2}$$

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TABLE 1.—DESCRIPTION OF VARIABLES

	Mean	SD	Comments
Dependent Variables			
Dogs	0.47789	0.70203	Number of dogs, more than 3 set to 3
Televisions	0.123330	0.94377	Number of televisions beyond the first; all have at least 1; more than 4 set to 4

Counts	0	1	2	3	4	5	6	7	8	9
Dogs	1992	931	205	48	8	3	1	0	0	1
Televisions	0	766	1282	772	263	71	24	7	4	0

	Mean	SD	Comments
Explanatory Variables			
Housing owned[a]	0.84980	0.35727	
Housing not owned	0.15020	0.35727	Rental: 0.13766, other: 0.01254
Housing duration			
0–2 years[a]	0.12324	0.32871	
3–4 years	0.14048	0.34749	
5–10 years	0.28379	0.45084	
>10 years	0.45249	0.49774	
Log (income/need)	1.01881	0.64842	Need equals poverty line for given household size; minimum: −1.83258 (income/need = 0.16); maixmum: 3.12895 (income/need = 22.85)
Male head			
None	0.24051	0.42740	Male head missing
PT/unempl	0.04139	0.19920	Part-time or unemployed
Out of lf	0.18125	0.38522	Out of the labor force
Male head's occupation			
Service[a]	0.32111	0.46690	Service, clerical, sales, private hh worker
Craft	0.17999	0.38418	Agriculture, operative, craft
Professional	0.25839	0.43775	Professional, technical, managerial
Male head's education			
<HS grad[a]	0.09721	0.29624	Education less than high-school graduation
HS grad	0.43807	0.49615	H.S. graduation, less than college graduation
College grad	0.22421	0.41706	At least college graduation
Female head			
None	0.03010	0.17087	Female head missing
PT/unempl	0.17686	0.38155	Part-time or unemployed
Out of lf	0.36939	0.48264	Out of the labor force
Female head's occupation			
Service[a]	0.72719	0.44540	Service, clerical, sales, private hh worker
Craft	0.02854	0.16650	Agriculture, operative, craft
Professional	0.21417	0.41025	Professional, technical, managerial
Female head's education			
<HS grad[a]	0.11258	0.31608	Education less than high-school graduation
HS grad	0.62496	0.48413	H.S. graduation, less than college graduation
College grad	0.23236	0.42234	At least college graduation

Notes: Total households: 3189. All explanatory variables, except the income-to-needs ratio, are dummy variables.

[a] Reference category not included in models.

The indexes are transformed into observables y_a and y_b with C discrete values from 0 to $C - 1$ and dummy variables d_{ij} $(i = 0, 1, 2, \ldots, C - 1; j = a, b)$, using, for each j, $C - 1$ parameters l_{ij} to divide the continuous standard normal into C regions. Of the $C - 1$ parameters l_{ij}, the first is normalized, $l_{0j} = 0$, and the $C - 2$ parameters from l_{1j} to $l_{C-2,j}$ are positive and monotonically increasing in i. We assume that each endogenous discrete variable has four possible outcomes in this note, but the number need not be exactly four and need not be equal for the two dependent variables. The number of theoretically possible values might not be the feasible number to consider in estimation, if some of the values do not occur or occur rarely in the data set. The problem is exacerbated in two dimensions where small cells interact. That is only a small sample problem, but it is a serious identification problem. The equations to define the observed y_j when $C = 4$ for both dependent variables are

$$y_j = 0 \quad \text{and} \quad d_{0j} = 1 \qquad \text{if } \epsilon_j \leq -z_j \tag{3}$$

$$y_j = 1 \quad \text{and} \quad d_{1j} = 1 \qquad \text{if } -z_j < \epsilon_j \leq l_{1j} - z_j \tag{4}$$

$$y_j = 2 \quad \text{and} \quad d_{2j} = 1 \qquad \text{if } l_{1j} - z_j < \epsilon_j \leq l_{2j} - z_j \tag{5}$$

$$y_j = 3 \quad \text{and} \quad d_{3j} = 1 \qquad \text{if } l_{2j} - z_j < \epsilon_j. \tag{6}$$

All $d_{ij} = 0$ unless otherwise stated.

The distribution of ϵ_a and ϵ_b in equations (1) and (2) is assumed to be joint normal, making this a bivariate ordered probit model. The variances of ϵ_a and ϵ_b are normalized to unity, and Cov $(\epsilon_a, \epsilon_b) = \rho$. If there are k parameters in $\boldsymbol{\beta}$, and there are A and B possible outcomes of the two dependent variables, there are $k + A - 2$ parameters in the first equation, $k + B - 2$ in the second, and $2k + A + B - 3$ in all, including ρ.

We now consider estimating either equation (1) or equation (2) by itself, so fix j. MLE is based on the probability of each possible value

TABLE 2.—GMM SPECIFICATION TEST RESULTS

Dog Equation
46 d.f. (3 sets of 24 orthogonality conditions minus 26 parameters): 49.66, $p > 33.0\%$
Television Equation
46 d.f. (3 sets of 24 orthogonality conditions minus 26 parameters): 67.90, $p > 2.0\%$
Joint Estimation of Dog and Television Equations
96 d.f. (53 orthogonality conditions from maximum likelihood plus 4 sets of 24 orthogonality conditions corresponding to dogs and televisions of 0 and 1, 0 and 2, 1 and 0, and 2 and 0 minus 53 parameters): 129.48, $p > 1.3\%$

of y_j. GMM estimation uses orthogonality conditions assuming explanatory variables are uncorrelated with the errors in predicting d_{ij} defined above. Given four possible values of y_j, there are $3k$ such orthogonality conditions; in general, there would be $(A - 1)k$ or $(B - 1)k$. Orthogonality conditions based on all of the d_{ij} would be perfectly collinear since all C of them add up to unity. For the case $C =$ 4, the orthogonality conditions are presented in the equations

$$E\{\mathbf{X}[d_{0j} - \Phi(-z_j)]\} = \mathbf{0} \tag{7}$$

$$E\{\mathbf{X}[d_{1j} - (\Phi(l_{1j} - z_j) - \Phi(-z_j))]\} = \mathbf{0} \tag{8}$$

$$E\{\mathbf{X}[d_{2j} - (\Phi(l_{2j} - z_j) - \Phi(l_{1j} - z_j))]\} = \mathbf{0}. \tag{9}$$

Note that ρ cannot be estimated in a univariate model. Let the vector of orthogonality conditions be $\mathbf{m}$. The univariate GMM estimator chooses the parameters in equation (1) or equation (2) to minimize $\mathbf{m}'\mathbf{m}$, estimates $V(\mathbf{m})$, then chooses the parameters in the same equation again to minimize $\mathbf{m}'[V(\mathbf{m})]^{-1}\mathbf{m}$. The resulting minimized value is distributed chi square under the null hypothesis that the orthogonality conditions are specified correctly, i.e., that the exogenous variables are uncorrelated with the disturbances and the functional form is specified correctly. The number of degrees of freedom in the test is the number of orthogonality conditions minus the number of coefficients estimated, here $3k - (k + C - 2) = 2k + C - 2$ in each equation. In

TABLE 3.—BIVARIATE ORDERED PROBIT ESTIMATED BY MLE

	Dogs			Televisions		
	Coefficient	SE	*t*-Value	Coefficient	SE	*t*-Value
Constant	−0.80989	0.16843	−4.80837	−0.64239	0.14564	−4.41097[a]
Housing						
Not owned	−0.24018	0.07083	−3.39099[a]	−0.29977	0.06389	−4.69186[a]
3–4 yr	0.03249	0.08614	0.37724	−0.10172	0.07784	−1.30671
5–10 yr	0.12309	0.07595	1.62076	0.09196	0.06939	1.32520
>10 yr	0.15474	0.07560	2.04681[a]	0.20687	0.06960	2.97232[a]
Income						
Log (income/need)	0.12973	0.04306	3.01256[a]	0.47798	0.03744	12.76600[a]
Household size						
2 in household	0.36080	0.09578	3.76713[a]	0.59919	0.08209	7.29947[a]
3 in household	0.76624	0.10378	7.38331[a]	1.02176	0.08984	11.37276[a]
4 in household	0.72750	0.10913	6.66640[a]	1.12200	0.09719	11.54392[a]
>4 in household	0.77465	0.11677	6.63409[a]	1.21248	0.10630	11.40607[a]
Male head						
None	0.01412	0.12128	0.11646	0.26374	0.10598	2.48845[a]
PT/unempl.	−0.19913	0.11957	−1.66537[b]	0.05311	0.10039	0.52899
Out of lf	−0.15926	0.08015	−1.98706[a]	0.00730	0.07224	0.10106
Craft	0.09363	0.07385	1.26781	−0.11591	0.06609	−1.75395[b]
Professional	−0.01887	0.06957	−0.27122	−0.00221	0.06201	−0.03571
HS grad	−0.09909	0.08299	−1.19396	0.14365	0.07528	1.90834[a]
College grad	−0.13014	0.09949	−1.30803	0.17501	0.08878	1.97121[a]
Female head						
None	−0.18963	0.18158	−1.04430	0.10654	0.15061	0.70742
PT/unempl.	0.00583	0.06377	0.09141	0.08185	0.05663	1.44539
Out of lf	−0.10524	0.05905	−1.78237[b]	0.03986	0.05241	0.76060
Craft	0.26345	0.12247	2.15113[a]	−0.12730	0.11457	−1.11109
Professional	−0.06630	0.06441	−1.02925	−0.03933	0.05725	−0.68705
HS grad	−0.08242	0.07653	−1.07693	0.02635	0.06989	0.37701
College grad	−0.09911	0.09460	−1.04765	−0.13301	0.08465	−1.57128
Ordered probit limits						
Limit 1	1.11709	0.03239	34.48483[a]	1.21407	0.02896	41.92645
Limit 2	1.82346	0.05415	33.67712[a]	2.14089	0.03822	56.01043[a]
Correlation of disturbances						
ρ	0.08715	0.02299	3.79013[a]			

Notes: Marginal impacts of the explanatory variables (all of the above except limits 1 and 2) on the probability of having a dog can be found by mulitplying the coefficients by 0.379889. Marginal impacts of the explanatory variables (all of the above except limits 1 and 2) on the probability of having a second television can be found by multiplying the coefficients by 0.245904. The marginal impact of the income-to-needs ratio is the coefficient times the appropriate factor divided by the income-to-needs ratio; so the coefficient itself applies at the poverty line (income-to-needs = 1.0).

[a] Significant at the 5% level.

[b] Significant at the 10% level.

Table 4.—Bivariate Ordered Probit Estimated by GMM

	Dogs			Televisions		
	Coefficient	SE	*t*-Value	Coefficient	SE	*t*-Value
Constant	−0.80175	0.16203	−4.94812^{a}	−0.60835	0.14293	−4.25636^{a}
Housing						
Not owned	−0.27535	0.06592	−4.17678^{a}	−0.28781	0.06253	−4.60248^{a}
3–4 yr	−0.02118	0.08118	−0.26095	−0.10185	0.07599	−1.34044
5–10 yr	0.09071	0.07316	1.23986	0.07677	0.06782	1.13192
>10 yr	0.10734	0.07292	1.47205	0.19694	0.06803	2.89513^{a}
Income						
Log (income/need)	0.11257	0.03913	2.87673^{a}	0.48690	0.03644	13.36232^{a}
Household size						
2 in household	0.40975	0.08773	4.67079^{a}	0.58068	0.08024	7.23697^{a}
3 in household	0.78159	0.09666	8.08572^{a}	0.99593	0.08747	11.38651^{a}
4 in household	0.75330	0.10253	7.34687^{a}	1.09638	0.09540	11.49303^{a}
>4 in household	0.81432	0.11039	7.37670^{a}	1.19325	0.10284	11.60313^{a}
Male head						
None	0.01457	0.11729	0.12420	0.23978	0.10429	2.29919^{a}
PT/unempl.	−0.25524	0.10951	−2.33078^{a}	0.07962	0.09771	0.81487
Out of lf	−0.14889	0.07766	−1.91730^{b}	−0.00892	0.07073	−0.12616
Craft	0.10929	0.07185	1.52119	−0.13463	0.06438	−2.09129^{a}
Professional	−0.05003	0.06797	−0.73616	0.00547	0.06084	0.08985
HS grad	−0.06362	0.08006	−0.79463	0.13593	0.07416	−1.83280^{b}
College grad	−0.08437	0.09556	−0.88294	0.16741	0.08714	1.92111^{b}
Female head						
None	−0.19032	0.16263	−1.17023	0.08263	0.14825	0.55733
PT/unempl.	−0.00039	0.06177	−0.00633	0.08004	0.05563	1.43869
Out of lf	−0.10516	0.05726	−1.83659^{b}	0.03425	0.05140	0.66641
Craft	0.21690	0.11728	1.84944^{b}	−0.09937	0.11241	−0.88400
Professional	−0.05207	0.06201	−0.83969	−0.05133	0.05564	−0.92253
HS grad	−0.07407	0.07454	−0.99378	0.04088	0.06873	0.59487
College grad	−0.10444	0.09130	−1.14388	−0.11065	0.08292	−1.33445
Ordered probit limits						
Limit 1	1.12824	0.03159	35.71664^{a}	1.22634	0.02814	43.58290^{a}
Limit 2	1.80105	0.05346	33.69140^{a}	2.14694	0.03718	57.73955^{a}
Correlation of disturbances						
ρ	0.12118	0.02163	5.60201^{a}			

Notes: Marginal impacts of the explanatory variables (all of the above except limits 1 and 2) on the probability of having a dog can be found by mulitplying the coefficients by 0.379889. Marginal impacts of the explanatory variables (all of the above except limits 1 and 2) on the probability of having a second television can be found by multiplying the coefficients by 0.245904. The marginal impact of the income-to-needs ratio is the coefficient times the appropriate factor divided by the income-to-needs ratio; so the coefficient itself applies at the poverty line (income-to-needs = 1.0).

[a] Significant at the 5% level.

[b] Significant at the 10% level.

general, the test has $(A - 1)k - (k + A - 2) = (A - 2)(k - 1)$, or $(B - 2)(k - 1)$, degrees of freedom in one equation. If $C = 2$, then there is no ordered probit, just binomial probit, and there are no extra orthogonality conditions, so the test requires ordered probit with at least three categories.

We now consider the joint estimation of equations (1) and (2), including ρ. The bivariate ordered probit model can be estimated by MLE or GMM. There are AB possible pairs of values of the dependent variables, or 16, with four possible values for each of the two dependent variables. MLE of the model uses the standard bivariate normal cumulative distribution function (cdf), defined over the AB regions determined by the limits l_{ij}. The $2k + A + B - 3$ first-order conditions lead to estimates for the $2k + A + B - 3$ parameters, including ρ.

For GMM estimation of the bivariate model, there are several sources of orthogonality conditions. The expectation of each product $d_{ia}d_{jb}$ ($i = 0, 1, \ldots, A; j = 0, 1, \ldots, B$) can be written as a bivariate normal probability. There could be as many as $(AB - 1)k$ orthogonality conditions from these expectations, since all explanatory variables are by the joint hypothesis uncorrelated with all AB of them; one pair must be omitted because all AB products add up to unity. The first-order conditions from MLE can also be used, since they are not perfectly collinear with the expectations of cell probabilities. Although both cell expectations and MLE first-order conditions are valid, small cell sizes and high collinearity can cause problems with convergence and inversion of the variance matrix. Cell probabilities can become quite small when two variables interact, and the linear independence of MLE first-order conditions from all cells together is a weak basis for estimation. Below the expectation terms are too collinear to identify the limits and correlation by themselves, and we report estimation using a subset of this choice set: the gradients of the log-likelihood function and the expectations of the cells corresponding to 0 televisions and 1 dog; 0 and 2; 1 and 0; and 2 and 0. To translate this into numbers, we have $k = 24, A = 4, B = 4$, 53 parameters, 53 first-order conditions from MLE, and 96 additional orthogonality restrictions.

Computer programs to estimate and test the ordered probit specification and to estimate the bivariate ordered probit model by MLE, for any number of categories, are available from the first author.

III. Data and Results

The discussion here is highly abbreviated; for more detail on the data set, see Butler and Chatterjee (1995). The data used in this paper are the demographic data concerning the households in a study of yogurt consumption. The households participated in a scanner panel data study supplied by A. C. Nielsen to the Marketing Science Institute. Data were gathered by the ERIM market testing service from 1985 through 1988. The data are from two test markets, Sioux Falls, South Dakota, and Springfield, Missouri, which are selected because they are demographically similar to the U.S. population as a whole. In each city, 2500 households were selected for the panel. The Household Demographics File covers the period between September 1985 and September 1988. To be included in the sample, a household must remain active in the panel and buy yogurt at least once in three years; 3189 did.

Table 1 summarizes the explanatory variables used in the model. The dependent variables have means of about 0.48 dog per household and 2.23 televisions per household. Every one of the 3189 households has at least one television, so the television dependent variable is the excess over 1.

The test results of the specifications of the univariate and bivariate ordered probit models are reported in table 2. The test of the overidentifying restrictions in the dog equation results in a chi square of 49.66 with 46 d.f., insignificant at any normal level of significance. In the television equation, the chi square is 67.90 with 46 d.f., significant at the 2.0% level. In the joint estimation, the chi square is 129.48 with 96 d.f., significant at the 1.3% level. If the generous level of significance of 1% is applied, then nothing is rejected, neither normality nor exogeneity. Alternatively, one could conclude that exogeneity is unlikely to be a problem, since it would affect both equations, but the functional form is questionable in the television equation. Perhaps the number of observations in the tails of the distribution (see table 1) of televisions is difficult to reconcile with a normal distribution.

GMM results under the null hypothesis should be, in theory, identical to those of MLE, because GMM is based on the MLE first-order conditions, which are sufficient for all of the parameters of the model, and other conditions. In a finite sample, given sampling error, the standard errors should be slightly smaller under GMM, and all 53 are.

We discuss briefly the estimation results. For more discussion concerning pets, see Butler and Chatterjee (1995). There are only small differences between the MLE results (see table 3) and the GMM results (see table 4). The principal difference is the larger correlation of disturbances under GMM (0.12 versus 0.08). The numbers of both dogs and televisions increase with home ownership and with tenure in housing. An increase in income relative to the poverty line increases the number of both dogs and televisions, but the effect is four times as large on televisions. Household size also increases both. More education for the male head of household and the absence of a male head increase the number of televisions, and males "around the house" (unemployed or out of the labor force) decrease the number of dogs. The correlation between the disturbances in the dog and television equations is positive and significant.

IV. Summary

This note proposes and uses a specification test of the normality and exogeneity assumptions on which univariate and bivariate ordered probit estimation is based. The ordered probit model implies expectations of various cell probabilities defined on the basis of the possible values of the dependent variable. The test is based on overidentifying assumptions in generalized method of moments estimation. As an example, the ownership of dogs and televisions by a sample of households is estimated as a function of economic and demographic variables. The tests do not reject the specification of the dog equation, but they do reject between the 1% and 5% level the specification of the television and bivariate equations.

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