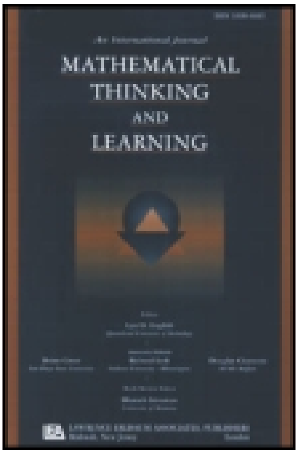

Mathematical Thinking and Learning

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/hmtl20

High School Students' Goals for Working Together in Mathematics Class: Mediating the Practical Rationality of Studenting

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Version of record first published: 17 Jan 2013.

Mathematical Thinking and Learning, 15: 24–57, 2013

ISSN: 1098-6065 print / 1532-7833 online
DOI: 10.1080/10986065.2013.738379

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High School Students' Goals for Working Together in Mathematics Class: Mediating the Practical Rationality of Studenting

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In this article I explore high school students' perspectives on working together in a mathematics class in which they spent a significant amount of time solving problems in small groups. The data included viewing session interviews with eight students in the class, where each student watched video clips of their own participation, explaining and justifying their behaviors. Analysis of data involved an investigation of students' *goals for working together*, which were found to vary along multiple dimensions. The dimensions that emerged from these data were mathematical versus non-mathematical goals, individual versus group goals, and personal versus normative goals. I present cases of four individual students to illustrate these dimensions. Such goals are important for illuminating how students' *practical rationality* is mediated by their personal goals for working together; additionally, these goal dimensions can be used as tools for considering challenges involved with using small group collaboration in high school classes where students' goals may be diverse.

Although there are examples in the literature where students collaborate and appear to develop positive attitudes toward working together (e.g., Boaler, 1998, 2002; Goos, 2004), promoting collaboration has often proven to be one of the most challenging aspects of reform teaching (Amit & Fried, 2005; Williams & Baxter, 1996; Wilson & Lloyd, 2000). Even in classes built around collaborative problem solving, with curricula providing open-ended tasks seemingly suited to group work and a teacher who believes in the importance of collaboration, students can be seen adopting noncollaborative roles when working together (Kotsopoulos, 2010). Why do students adopt these roles?

Activity in mathematics classrooms, including small group collaboration, has often been studied at the group level, where the objects of inquiry are social phenomena such as discourse or norms (Bishop, 2012; Cobb & Yackel, 1996; Goos, 2004; Gresalfi, Martin, Hand, & Greeno, 2008; Wood, Williams, & McNeal, 2006). Other research places emphasis on trying to understand the classrooms through the eyes of individual students (Boaler, 2002; Cobb, Gresalfi, & Hodge, 2009; Herbst & Brach, 2006; Jansen, 2006, 2008; Walter & Hart, 2009). Each of these approaches reflects a different focus; one is primarily concerned with the social factors that impact behavior, whereas the other is more concerned with cognitive or psychological factors. However,

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activity in a mathematics classroom involves both social and cognitive/psychological factors, and both deserve attention (Cobb, 1994).

One framework in which both of these factors are acknowledged is *practical rationality*, a construct originally used to frame teacher decision-making (Herbst, 2010; Herbst & Chazan, 2003). Practical rationality is described as a collection of *shared social resources*, including instructional norms and obligations to stakeholders that together define a set of possible teacher actions. These resources are mediated by teachers' *personal resources*, individual traits that include beliefs, knowledge, and goals. It is this through this process of mediation that eventually teachers decide on particular actions.

In her dissertation, Aaron (2011) adapted this framework to describe the decision-making of *students*. This framework is particularly useful because it incorporates both social and individual factors in describing how particular student actions come to take place in mathematics classes. Her study focused on expanding and delineating the impact of some of the social factors influencing student decisions, particularly norms. In contrast, the study described in this article attends to a particular personal factor: students' goals for working together. This research adds a layer of detail to the practical rationality framework, and because it focuses on students' perspectives on working together, it also provides a new tool to make sense of one of the more challenging mathematics teaching practices—promoting collaboration.

The study took place in a high school class where communication and problem solving were prominently featured, and where a small group learning format was used on a daily basis. I visited this class 23 times over the course of a semester, and interviewed eight students in this class twice. In the second interview, I incorporated a *viewing session,* where each student was shown video clips of his or her own participation in a small group learning episode and was asked to interpret and explain his or her actions. In this article I focus on four cases in the data set in order to illustrate three dimensions of students' *goals for working together* that emerged in the analysis of their interviews. In the discussion, I describe possible implications of these goal dimensions types for mediating the social factors defined in practical rationality.

RESEARCH QUESTIONS

The initial research questions guiding the study were, "In a high school mathematics class where students are regularly asked to work together to solve mathematical tasks, what behaviors do students self-report as acceptable, desired, or appropriate? What rationales do they provide for these behaviors?" As data were collected and analyzed, students' goals emerged as a research focus, as students provided goals more regularly than other rationales for their behavior (such as the actions of the teacher). Therefore, a more focused research question was posited: "Given these described rationales, how can students' *goals for working together* be characterized?"

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Working Together

In this article, I use the phrase "working together" to refer to situations in which students are organized into small groups of two to four and are tasked with solving a mathematical problem.

I purposefully do not use the term *collaboration*, as students can work together in ways that might not be considered collaborative—that is, they may not lead to increased mathematical knowledge or discourse in the group, or to equitable distribution of agency or understanding (e.g., Clarke & Xu, 2008; Esmonde, 2009c; Kotsopoulos, 2010; Williams & Baxter, 1996).

There has been a substantial amount of research on how cooperative behavior might be promoted in small groups in mathematics classes (Cohen, 1994; Fuchs et al., 1997; Goos, 2004; Jansen, 2012; Slavin, 1996; Steinbring, Bussi, & Sierpinska, 1998; Webb & Farivar, 1994; Webb, Nemer, & Ing, 2006). Researchers have highlighted the importance of the task (Cohen, 1994), the scaffolding and modeling provided by the teacher (Webb et al., 2006), accountability structures (Slavin, 1996), and encouragement and valuation of student competence (Boaler & Staples, 2008).

Despite this research, Walshaw and Anthony (2008) pointed out that productive student discourse is not caused by teaching; while it may be *occasioned* by certain teaching practices, students' beliefs, goals, and attitudes play a role in how discourse plays out in the classroom.

Part of the challenge of using collaborative small groups is that students may adopt different roles, often inequitable, when working together (Esmonde, 2006, 2009c; Kotsopoulos, 2010; Sinclair, 2005). Esmonde (2009b) described how roles such as *expert*, *novice*, *in-between*, and *facilitator* get negotiated differently in different kinds of class activities, such as preparing a class presentation or taking a group quiz. This work emphasizes the highly contextualized nature of group work and illustrates how students' identities and actions are situated within and constrained by instructional settings and the resources available to them (Esmonde, 2009b; Webb et al., 2006).

Social Resources

Some of the resources available to students are *social resources*; they exist as a result of participation in a particular context or community. An example is sociomathematical norms, patterns of behavior that are interactively constituted through activity in a classroom and that are distinctly both social and mathematical (Cobb, Wood, Yackel, & McNeal, 1992; Cobb & Yackel, 1996). Sociomathematical norms establish boundaries for how mathematical activities such as explaining or justifying are determined and acted on by the participants, and influence the nature of the explanations and justifications that are expected by students (Weiss, Herbst, & Chen, 2009). Students in different classes are able to make use of the resources that are provided by their particular context by, for instance, pressing a classmate to provide an explanation in addition to an answer. This is possible because the sociomathematical norms in the class legitimize and encourage this kind of move, while the norms in another class may not.

Some researchers have investigated how other social resources, such as the explaining practices modeled by the teacher (Webb et al., 2006), the distribution of mathematical authority (Amit & Fried, 2005; Cobb et al., 2009), and the construction of mathematical competence (Gresalfi et al., 2008), can influence collaborative activity in the classroom. For example, Gresalfi and colleagues (2008) investigated how competence was constructed ("made meaningful") in the context of two middle school classrooms. Competence was defined as a system which included the agency students were afforded by the participation structure established in the classroom, as well as how accountability was established (what students were accountable for doing, and to whom they were accountable). The researchers documented "the affordances of various aspects of the classroom systems, and the ways that these affordances were taken up (or not) by various participants" (p. 56). While this framing of competence focuses attention on students' interactions with each other and the teacher, it leaves unexamined students' interpretation of their interactions—in particular, how the *students* defined what it meant to be competent in their mathematics class. It also downplays the possibility that different students might define competence differently.

This work makes it clear that social resources are important factors that influence the nature of mathematical activity when students are asked to work together. But it also points to a need to understand why student do not always make decisions that are in line with the norms established in the classroom (Levenson, Tirosh, & Tsamir, 2009). One explanation is that social resources are not the only resources available to students. They also bring with them personal resources which influence their decisions.

Personal Resources

In contrast to the previous research in which social resources are the object of inquiry, many researchers have explored the perspectives of individual students who participate in social activity in mathematics classrooms (Boaler, 2002; Esmonde, Brodie, Dookie, & Takeuchi, 2009; Herbst & Brach, 2006; Jansen, 2006, 2008; Kotsopoulos, 2007b; Levenson et al., 2009; Peterson & Swing, 1985; Walter & Hart, 2009). Levenson, Tirosh, and Tsamir (2009) found that students in interviews valued explanations that were not valued by the teacher or valued in classroom interactions. They claimed that the *perceived* sociomathematical norms were different than the norms enacted in the classroom, a finding which highlights the need to explore students' perspectives as well as their actions.

In one example of such work, Peterson and Swing (1985) found that the beliefs of elementary students were related to the quality of small-group learning. Specifically, students who said that they valued problem-specific explanations were more likely to engage in providing or receiving higher order explanations, while students who valued general or procedural explanations were less likely to engage in listening behaviors and more likely to engage in answer-checking. In another example, Jansen (2006) found that some middle school students were more likely to participate in conceptual discussions in their mathematics class if they felt that their participation could help their classmates understand. In this case, students' concern for their classmates was a personal factor that influenced their decisions about participating in collaborative activity.

This research emphasizes the importance of understanding not only the social resources that influence student behaviors but also the personal resources that individual students bring with them into the classroom. Investigations into these resources can go beyond showing whether students adopt particular norms, but also can also reveal *why* they adopt these norms (Cobb et al., 2009). This work could also attend to the range of personal resources that could be at play in a single classroom. There are few frameworks which acknowledge the influence of and interplay between personal and social resources. One such framework is *practical rationality*.

The Practical Rationality of Studenting

Practical rationality was originally developed to explain the decision-making of teachers (Herbst, 2010; Herbst & Chazan, 2003). It acknowledged that teachers take on professional roles in established institutions, and that these roles come with built-in affordances and constraints. Instructional norms and obligations to stakeholders are social resources that constitute both boundaries on the work of teaching as well as affordances for accomplishing this work. At the same time, teachers bring personal assets with them—goals, beliefs, and knowledge—which enable some teachers to do things other teachers cannot. Teachers justify their actions sometimes on the basis of personal grounds and sometimes on the basis of professional/normative grounds. There is interplay between the two kinds of resources, and Herbst (2010) argued that teacher decision-making can be better understood and supported if room is made for both.

Drawing on and modifying this work, Aaron (2011) applied the practical rationality framework to students, describing the "work" that students do as *studenting.* Studenting includes academic learning, but it also acknowledges that students balance many kinds of obligations and goals as they go about the work of being a student. They draw on both personal and social resources in order to justify their decisions and actions.

The diagram in Figure 1 shows the interplay between these resources. In the framework, the practical rationality of studenting consists of two social resources: the *norms* implicit in the classroom and students' *obligations* to various stakeholders (see Figure 1).

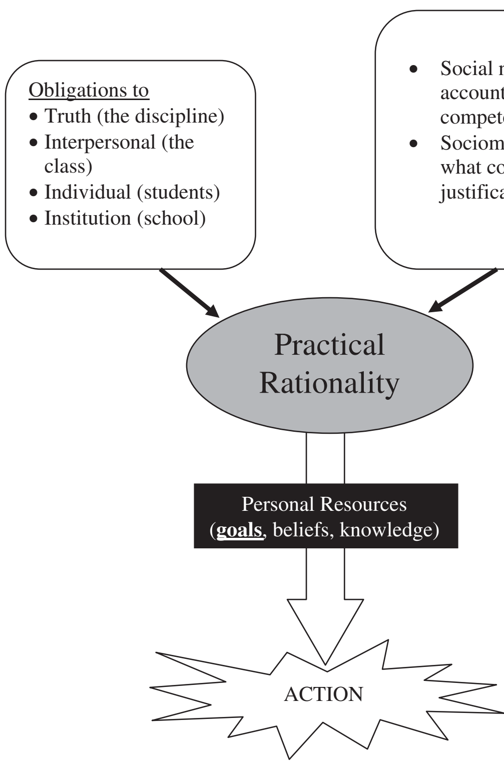

FIGURE 1 The practical rationality of studenting (adapted from Aaron, 2011. Used with permission).

Norms are the "oughts" of mathematics classrooms, social resources which are constituted in the classroom and which guide behavior and expectations. Aaron (2011) identified several studenting norms in geometry, including "students ought to share ideas when they are different from the ideas of others" and "students ought to complete ideas that are incomplete."

Obligations are social constraints on the work of studenting, which come from four external sources: general interpersonal relations in the classroom, other specific individuals, the institution of school, and truth. These shared social resources are often in competition with each other as students make decisions and act. For instance, when a classmate shares an incorrect answer, a student may want to question the solution due to a concern with knowing what is correct (an obligation to truth). She may not want to correct her classmate to avoid embarrassing him (an individual obligation to that classmate). Interpersonal obligations might lead her to want to correct the classmate out of concern that the other students in the class know the correct answer, or they may lead her to refrain from correcting because she does not want to create a situation of discomfort for the rest of the class. All of these are examples of external obligations that might be at work, perhaps all simultaneously.

The student in the situation just described must eventually make a decision about correcting the classmate with the incorrect solution. In order to make this decision, the perceived obligations are filtered through her personal resources (see Figure 1). While norms and obligations are social resources that are available to all students as a result of participation in a mathematics class, personal resources are "characteristics or traits of individuals that are unique to specific individuals in a particular context" (Aaron, 2011, p. 259), including students' personal goals and beliefs. In the previous situation, if the student has an underlying goal orientation in which mastery and understanding is valued, the obligation to truth might win out over individual or personal obligations, and she may correct her classmate.

Students' Goals

Goals are one of the personal resources that mediate the social resources available in the classroom. Goals are defined as "cognitive representations of what individuals are trying to do or what they want to achieve" (Pintrich, Conley, & Kempler, 2003, p. 321). They are particularly important objects of study because they reveal underlying motivations for student behavior (Walter & Hart, 2009).

Previous work on goal orientations has identified essentially three fairly stable goal types: performance-approach (demonstrating competence and outperforming others), performance-avoidance (avoiding failure in front of others), and mastery/learning (improving competence) (Ames, 1992; Pintrich et al., 2003). However, these orientations are often too broad for investigating specific classroom practices. They do not take into account the possibility that goals might be contingent on subject matter or the specific activity in which a student is engaged. This is of particular importance in the current study, because it explores how different types of goals operate with regard to a particular practice (working together) in a particular mathematics class.

Recently, researchers have made efforts to expand the dimensions of students' goals in mathematics classes in order to investigate the relationships between various types of goals and provide new ways of thinking about how goals are contextualized in particular settings (Jansen, 2006; Walter & Hart, 2009). For example, Walter and Hart (2009) found that calculus students in a collaborative learning environment were motivated by intellectual-mathematical and social-personal goals in tandem.

Goals for Working Together

In Slavin's (1996) review of research on collaboration in mathematics classes, he argued that teachers need to provide both group goals and individual accountability in order to promote effective collaboration. Individual accountability is needed so that students must all contribute, and group goals are needed so that students have incentives to help each other. However, the work cited by Slavin attended to accountability and goals as envisioned and promoted by the teacher, not as seen through the eyes of students.

There is some research that investigates indirectly the relationship between goals and collaborative group work. Cohen (1994), in describing Complex Instruction (CI), emphasized the importance of using group tasks that cannot be solved by any member of the group individually, so that solving the problem becomes a task that necessitates input from multiple members of the group. Boaler and Staples (2008) found that high school students in CI classes expressed responsibility not only for meeting individual goals, but also for helping their classmates understand. However, there is also research that suggests that students working in groups may see their group interactions *only* as opportunities to meet their own individual goals, by treating their peers as experts who can give them the solutions (Amit & Fried, 2005; Kotsopoulos, 2007b).

These contradictory findings reveal a need to investigate the impact of personal resources on behavior in small group work. In particular, research should attend to students' goals specifically related to their own group work practices. Because the teacher does not directly supervise all of the students during group work, the practice of working together may provide increased opportunities for personal goals to drive decisions and behaviors. Some students might mostly want to get the answers and proceed through the tasks, while some may want to socialize, while some might want to understand the material, and other might simply want to be left alone. The diversity of students' goals may explain the development of inequitable distribution of mathematical expertise and agency found in some recent studies (Esmonde, 2009c; Kotsopoulos, 2007a, 2007b, 2010).

There is little research that explores the diversity of goals within a particular mathematics class. Even research that attends to the perspectives of individuals tends to make claims at the class level (Boaler, 1998, 2002; Boaler & Staples, 2008; Cobb et al., 2009; Jansen, 2012). Teaching mathematics through small group work means attending to a potentially wide range of different goals and providing a flexible structure whereby students are motivated to engage in mathematical discussions with each other and reach similar mathematical conclusions. Understanding students' goals might provide teachers with the leverage they need to establish this structure.

Summary

The research described in this article adds to the literature by defining dimensions upon which high school students' goals for working together might vary in a particular mathematics class. It shows that there can be substantial diversity of goals in a single class and, by including students' voices, sheds light on the challenges teachers face in promoting group work amidst this diversity. Furthermore, the goal dimensions constitute a tool with increased sensitivity for identifying and describing goals in relation to a particular practice in a particular context, as opposed to broad goal orientations. This is important because goals are likely dependent on context. This study is also somewhat unique in that I used viewing session interviews (described in the next section) to investigate students' goals in an attempt to limit the weaknesses of self-report data. Finally, the conceptual framework employed in this article allows for conjectures about the mediating function of each student's goals in filtering shared social resources. This is important because many studies attend primarily to either individual or social resources and ignore or downplay the role of one or the other.

METHODS

I used a case study approach to address the research questions, treating each interviewed student as a case within the bounded system of the mathematics class under investigation (Yin, 2008). "Working together" is also considered a boundary on the system being investigated (the analysis of data presented in this article is restricted to descriptions of small group work).

The research design included the collection of multiple data types, both observations and interviews. By asking students to describe their goals while watching videos of their own interactions, I triangulate between these data sources (Clarke, 1997; Yin, 2008). I employed a grounded theory approach (Charmaz, 2006) to identify themes that were not preconceived.

Participants and Context

The school

The high school where the study was conducted was located in a vocationally focused public school district in the Mid-Atlantic region of the United States with an enrollment of approximately 1500 students. In addition to traditional high school subjects, the school offers courses in 42 different career paths grouped into six career clusters, including business, construction, and health services. The emphasis on vocational training is emphasized particularly in 11th and 12th grade, when students are given access to technical training and specialized skills in their career area.

One of the distinguishing features of the school's approach to mathematics teaching was the use of the *Core Plus Mathematics* curriculum (Hirsch et al., 2008). Importantly, these materials include investigations where students are expected to collaborate to solve problems for which solution strategies are not already known.

The teacher

Mr. Neal (this and all other names are pseudonyms) was chosen to participate in the study on the basis of my previous observations of his classroom (Webel, 2010b). While his instructional practices and views about teaching are not treated as variables in this study, they are an important part of the context in which the research was conducted and will be described briefly in the results section. Mr. Neal had been teaching at the school with the Core Plus Mathematics curriculum for three years.

The class

One of Mr. Neal's Integrated Math 3 classes was selected based on his preference. It was an untracked class (students were not divided into "honors" and "regular" sections). Of the 24 students in the class, 22 agreed to participate in the study. Seventeen of the 22 students were in 10th grade, four were in 9th grade, and one was in 12th grade.

Target students

A subset of eight students was selected to be interviewed based primarily on results of a student survey, which was given during the sixth week of class. The survey posed questions designed to reveal students' perspectives about working together. The survey was used to recruit participants who were similar in that they were all in the same mathematics class, but also were likely to hold different views about working together, which is consistent with Merriam's (1988) recommendation that cases "be selected for their power both to maximize and to minimize differences in the phenomenon of interest" (p. 154). Also, as case study research often serves its major purpose (contributing to theory) by attending to atypical cases (Yin, 2008), the selection process was not designed to generalize to the population in Mr. Neal's class or to other high school mathematics classes. It does, however, allow some insight into what it meant to use group work in the context of Mr. Neal's class, as these target students were participants in that context.

The survey contained items that probed the extent to which students valued a variety of behaviors when working together, such as sharing answers, explanations, and justifications, reaching consensus on a problem before moving on, and understanding versus completing tasks. I selected four students who seemed to value the collaborative construction of knowledge (sharing justifications, evaluating ideas of peers, etc.), three students who did not seem to value these behaviors, and one student who had a mixture of views according to the survey but was observed to be very engaged in class discussions. These initial categorizations were rough, as dimensions of students' goals had not yet been developed. However, the process allowed me to investigate cases of students with differing perspectives about working together.

Data Collection

Class observations

I observed 23 90-minute class sessions over the course of the semester, taking fields notes and making video recordings of each entire class session. I observed the first eight classes of the semester, and then visited approximately once a week for the next 12 weeks. All of the observation data were collected between January 20 and April 27, 2009.

During whole class discussions, I set the camera in the back of the room so that the chalkboard and many students were visible. During periods of group work, I focused the camera on a specific group, using the same group over the entire lesson even if there were multiple episodes of group work. One of my goals initially was to ensure that students were accustomed to the camera so that it would be less likely to influence their discussions. Once the survey was given and target students were selected, the camera was always positioned to record the group interactions of one or more target students. Mr. Neal typically used group work multiple times during each class session, which allowed me to capture more than 30 episodes of group work.

Interviews

I interviewed Mr. Neal at the beginning of the semester. This interview was primarily conducted to provide contextual information about Mr. Neal's perspectives on using group work and some of the rationale behind his practices.

The primary focus of this study was the student interviews. I interviewed each of the target students twice. The first interview was conducted with each student after school between March 9 and March 23. This interview took approximately 45 minutes.

The second interview, conducted immediately after observations were concluded (between April 27 and May 8), used viewing sessions to help students explicitly ground their perspectives in the context of their participation (Kotsopoulos, 2007a; Peterson & Swing, 1985). In this interview, I showed each of the target students one or two clips of their participation in an episode of group work, asking them questions about their participation (see the Appendix for one example of an interview protocol). I transcribed the dialogue from the clips and provided students with the transcriptions to refer to as they watched the clip. When appropriate, I provided students with the math textbook turned to the page with the problems from the clip.

I chose clips based on their potential to elicit information about the students' perspectives about working together. Like Kotsopoulos (2010), I found that students' descriptions of their behavior sometimes did not align with observed behaviors. For example, a student could say that they press their classmates for explanations in addition to answers, but the video might reveal the student appearing to copy answers. I chose clips with such disconfirming evidence for use in the second interview in order to (1) challenge my initial assessment about students' goals and (2) provide them an opportunity to revise their positions in light of the videotaped data, which they often did by qualifying their original positions. In cases where the video clips did not provide disconfirming evidence, I selected clips that highlighted other aspects of the students' participation in order to prompt further elaboration about their perspectives on working together. The clip lengths ranged from about two minutes to about five minutes, although longer clips were usually played in segments during the interview.

I refer to this method as *viewing session interviews*, a form of what Kotsopoulos (2007a) described generally as *video study methodology*, which treats participants' perspectives on their video performances as central. Viewing session interviews are different than *stimulated recall interviews* (Anthony, 1994; Clarke, 1997), in which participants are asked to reconstruct their cognitive processes, feelings, thoughts, etc., while watching a tape of their participation. One of the criticisms leveled at this approach is that the decision-making employed during the watching of a video is different than the decision-making employed during the events themselves (Barnes & Todd, 1978; Lyle, 2003). Therefore, when a subject is asked to recall their cognitive processes during the recorded events, a bias is introduced.

The primary purpose of the video in this study was not to have students reconstruct the cognitive activity that occurred during the events on the video, but to encourage reflection by providing visual access to a context in which the activity in question—working together—was enacted. The goal was not necessarily to know what students were thinking *during* the episode, but rather to investigate how students think about the way they participated in retrospect. While I occasionally asked students at times to try to remember what they were thinking at specific recorded moments, accurate reconstruction of past thoughts and feelings is not relevant to my analysis. My focus was at a larger grain size—how do they presently rationalize and/or explain the past activity in question? In this I follow the recommendations of Lyle (2003), who suggested that the use of video-based interviews in educational settings should “favour the ‘stop and remember’ rather than ‘talk you through it’ approach” (p. 873).

The viewing session interview improved validity because it reduced the extent to which the constructed themes were guided solely by students’ descriptions of behaviors, which could be influenced by their views about what is a desirable response and my position as a teacher-like adult (Kotsopoulos, 2007a). It improved the connection to context and also ensured that there were multiple kinds of data to inform the development of each case (Yin, 2008).

Analysis of Data

Small group discourse

The clips shown to students were analyzed using the framework established by Esmonde (2009c). I identified the roles adopted by students as *expert* (one who is granted authority to determine the correctness of solutions), *novice* (one who defers to the expert), *in between* (neither expert nor novice), and *facilitator* (one who orchestrated group activity and solicited input from group members). Like all researchers who analyze speech, I acknowledge that these assignations are best guesses, and represent attempts to, in the words of Barnes and Todd (1978), “reconstruct from our understanding of the context the meaning most likely to have been understood and acted upon by other participants” (p. 107). However, because I showed participants the clips and used *their responses* as the primary data source, inferences about student roles are not central to the major claims in this paper, but merely serve as context which aid in interpreting students’ goals for working together.

Interviews

This research incorporated an iterative process of data collection, memo writing, and analysis (Charmaz, 2006). After the first semistructured interview, I transcribed each interview in its entirety. Then I performed a preliminary within-case analysis for each target student, identifying preliminary themes for use in selecting video clips for the second interview.

After the second interview was conducted and transcribed, these new data were coded and combined with the data from the first interview in order to create a profile for each student. Because this article is focused on small group work, I have not included data where students specifically talked about participation in whole class discussions.

At each stage the data were analyzed and re-analyzed using qualitative analysis software. I used an open coding approach, attending to meaning rather than form for student utterances. By focusing on meaning, I was able to take into account the context in which statements were made, attending to fluid and dynamic features of our conversations rather than the counts of particular words or structures. This approach necessitated inferences about students' meaning, but this is a constraint in any study in which speech is analyzed (Barnes & Todd, 1978). In the results section I describe how I attributed meaning to students' statements and attempt to provide enough examples to allow the reader to judge the strength of inferences.

On the first pass through the data, I identified incidents of described actions or behaviors on the part of the student, both hypothetical and recollected. The goal was to capture the different kinds of behaviors that each student considered to be acceptable, desired, or appropriate. Each incident was given a concise code that captured the essence of the behavior. For example, a statement such as, "I like working in groups because I can get help from others" was coded as "getting help." Even though the student is not describing an incident of getting help, he or she implies that getting help was a behavior that was desirable, acceptable, or appropriate. These were constantly compared as they were developed, eventually resulting in 18 codes, including "help others," "press for explanations in addition to answers," and "give up on working together."

It became apparent that coding incidents of described behavior did not capture crucial differences between students. For example, a student might say that they engage in helping others, but might qualify this by specifying that they only help their friends, only help when they are confident in their solution, or only help when other students ask for help. Likewise, they might provide different justifications for the behavior, such as helping because they might need help later, or helping because of a genuine desire to contribute to someone else's understanding. Each of these qualifiers gives a different sense of the underlying goal behind the behavior. On the second major pass through the data, I went through each coded behavior and searched for qualifying statements attached to behaviors, which included *reasons why* or *conditions under which* students would engage in a behavior. Not every behavior was qualified, and some behaviors had multiple qualifiers.

For each student I created a research memo where all of the roles and accompanying qualifiers were consolidated and summarized (Corbin & Strauss, 2008). Figure 2 shows a hypothetical example of a concept map generated from a research memo. Behaviors are represented by rectangular boxes, while qualifiers are represented by ovals.

After all of the cases were analyzed individually, I performed a cross-case analysis to identify differences in the eight profiles. The differences that emerged seemed attributable to different goal orientations; that is, different students were trying to achieve different things. Not only were their

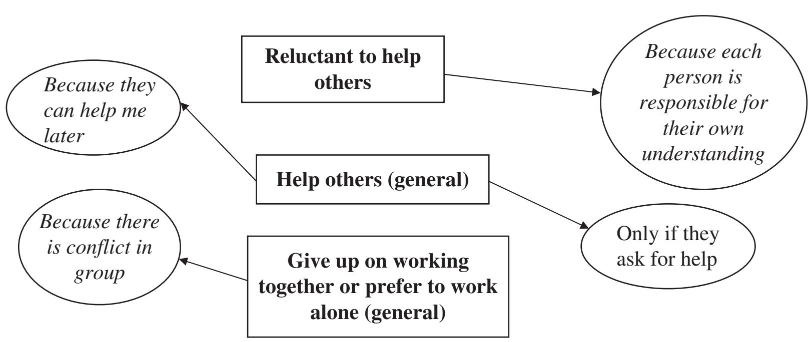

FIGURE 2 An example of behaviors and qualifiers. *Reason* qualifiers are italicized, and *condition* qualifiers are not italicized.

goals different but they also located responsibility for achieving them differently, and in some cases their goals seemed to originate from different sources. The three dimensions of students' goals for working together described in the remainder of this article represent my attempt to delineate and communicate the essence of these differences.

RESULTS

In the following sections I first share some data about the context of Mr. Neal's class, and then I describe the cases of four students to illustrate the different dimensions of students' goals that emerged in the interview data. Each described case was selected purposefully to highlight a particular aspect of studenting—working together—and some aspects of the practical rationality—goals for working together—that accompanied it. In the discussion section, I describe conjectures about how students' goals mediated social resources in each case.

Mr. Neal and His Teaching

It was clear from talking to Mr. Neal that he valued collaboration as a way of promoting mathematics learning. During his interview he talked about the role of the class community.

> . . . when we do work in groups, the goal is to have as many thoughts and opinions and ideas as possible so that together, collectively, that group or the whole class can determine what it is that is correct. What is true. What is right versus what is not. As opposed to me saying this is right, this is not. The goal of them is to come up with that.

My interview with Mr. Neal suggested that he believed in the importance of students collaborating to solve problems without relying on his authority.

In Mr. Neal's classroom, the lesson typically started with a warm-up exercise (usually related to the material from the day before or previewing the upcoming lesson), completed individually, followed by a whole class discussion about the warm-up. Mr. Neal then introduced a new activity from the textbook, followed by a period where students worked on problems in groups, coming back to whole class discussions periodically. During group work, Mr. Neal circulated among groups and provided assistance, usually a combination of mathematical and social scaffolding (Baxter & Williams, 2010). He rarely told students whether their solutions were correct during group work. During whole class discussions, Mr. Neal asked students to present their solutions, sometimes by simply putting their answers on the board, and sometimes by explaining their solution strategies. Often, Mr. Neal would follow up their explanations with explanations of his own, and during these explanations he often resolved questions about the validity of a solution.

Nonmathematical Goals Trump Mathematical Goals: The Case of Logan

Logan voiced some mathematical goals that implied a desire to better understand mathematics (for example, he claimed that he wanted others to explain their answers so that he could be convinced that they were correct). But in many cases he seemed more motivated to engage in behaviors out of a desire to complete tasks, to socialize, or both ("we try to spread out our time so we can have a conversation and still get the work done"). One of the interesting features of Logan's case was the extent to which his social concerns overshadowed his mathematical concerns, as in the following dialogue:

1 Logan: I like working in groups, it just depends on who I'm working with.
2 I: Okay.
3 Logan: Teachers say it doesn't work to work with friends, but I get my work done when I'm working with somebody I like. When I'm with somebody I don't like, I'm not going to do the work.
4 I: Okay, so when you're working with someone that you like, why is that good?
5 Logan: 'Cause I'll do the work, I'll help them. I won't help the other person.

In this excerpt, Logan indicates that his relationships dictate whether he will help the other members of his group (turn 3) (behavior *help others*, condition *only if I'm in a group with my friends*). When Logan talked about working with others on mathematics, he nearly always referenced relationships as motivating his behavior, even to the extent that mathematical validity was set aside. For instance, he indicated that he would refuse to agree with a classmate not because of the mathematical content of their idea, but because of his relationship with them.

> . . . if you don't like that person, you're not agreeing with their answer. If I, if had a fight with that kid, and me and him got into it the other day, I'm not agreeing with his answer whether he's right or not. (Behavior *solving problems together*, condition *only if I'm in a group with my friends*.)

Even when reading a written vignette presented during Interview 1 (see Appendix), Logan said he would prefer Jane's help (a procedural explanation) apparently for nonmathematical reasons. "I just don't like Jane's because she's . . . got an attitude. . . . Like, she's saying, I'm right, I'm better than you.' But you're not. The other way is better." In this example, Logan ignores the mathematical content of the hypothetical explanations, instead focusing on Jane's "attitude" (behavior *get help from others*, condition *only from someone I like*).

Logan's viewing session

In one of the episodes I watched with Logan during his viewing session interview, he was in a group with Lindsay and Sevanye. Logan joined the group late, after Sevanye had done most of the problem, while Lindsay appeared to copy Sevanye's work (this episode is described in detail later in this article). When Sevanye was confused, she turned to Mr. Neal for help without asking Logan or Lindsay to contribute. During Mr. Neal's conversation with Sevanye, Logan and Lindsay began their own nonmathematical conversation. Almost all of the mathematical talk during the episode came from Sevanye, and during the viewing session interview, Logan claimed that he intended to copy Sevanye's answers. In Esmonde's (2009c) terms, Logan's role might be characterized as a *novice*, one who deferred to others' mathematical authority. He also admitted that he did not learn any new mathematics during the episode, claiming that it was review. When I asked him to name factors that make group work more or less productive, he again brought up relationships:

1 Logan: Yeah if I don't like the person that I'm in the group with. Like if you sit next to Cindy, oh god I don't want to work with her. I don't want to sit next to her, I don't want to work with her.
2 I: But you liked this group?
3 Logan: Yeah, I like Lindsay. I don't like Sevanye.
4 I: But you didn't really work well with her?
5 Logan: Who, Lindsay? No I like working with Lindsay.

6 I: But you didn't work very well with her.
7 Logan: No we were just talking. Like I said me and her were kinda bored and preoccupied in conversation.
8 I: As a teacher, if I wanted you to learn more, I probably wouldn't put you with Lindsay, would I?
9 Logan: No, you would. To be honest I don't like when the teacher chooses people that he knows we're not friends. That just, that just makes me mad. Why, why am I gonna do the work if you're gonna put me with somebody that I can't even work well with?

Again, in the group work and in the interview we see Logan's emphasis on his nonmathematical goals, which have to do with *not being bored* (turn 7), *working with people he likes* (turns 1, 3, 9), and *socializing* (turn 7). Even though he admits that this episode is not productive mathematically, it does meet his social goals. He makes the argument that he works better when he works with his friends, but in this case he appears to be conflating "working well" with enjoying working in a group (turn 9, behavior *solving problems together*, condition *only when working with my friends*). He seemed confused when I suggested that he didn't work well with Lindsay (turns 4, 5). Because his social goals dominate his perspective, he discounts the low mathematical quality of this episode of group work, and seems to be satisfied with how the situation played out even though he admits that he did not learn very much.

Mathematical and nonmathematical goals

Logan's case shows how nonmathematical goals (and especially social goals) can take precedence over mathematical goals and create tension around the studenting practice of working together. Unlike other students in the study, Logan's approach to working together seemed nearly entirely unmotivated by the goal of building mathematical understanding. Table 1 shows a consolidated list of Logan's mathematical and non-mathematical goals along with the associated behaviors and qualifiers.

Individual Goals Versus Group Goals: Sevanye and Connor

Sevanye's and Connor's goals

In contrast to Logan, Sevanye and Connor seemed to be more motivated by mathematical goals. When comparing these two students, a prominent contrast was not the content of the goal (what they were trying to achieve) but rather the locus of responsibility for meeting their goals. In the following sections I provide examples of several of their described behaviors and qualifiers and explain how these constitute evidence for the indicated goal types (individual or group).

The following quotes from Sevanye were coded under the reason qualifier *because each person is responsible for their own understanding*:

> It's up to everyone . . . like I fall asleep sometimes in class, and I miss stuff, but it's my fault for falling asleep, I can't expect anybody else to carry me 'cause I don't know what I'm doing. That's not fair to anybody. Especially if they know what they're doing 'cause they paid attention.

> If you don't want to do your work, I'm not going to say anything to you. We are all in high school, graduating in two years. If you can't be smart enough to know I need to get this work done 'cause there's a test next week then . . . that's not my problem.

TABLE 1
Logan's Goals

Goal Type	Goal	Behavior *(Qualifiers)*
Nonmathematical	Completing tasks (getting answers)	Get answers only (*No assessment soon—just want to get the work done*)
		Get help from the teacher (*Because I want to know if the answer is right*)
	Socializing	Socializing (*Because the math is too easy, because the math is too hard*)
		Solving problems together (*only if I am in a group with my friends*)
	Getting help (general)	Get help from others (*Only from someone I like, only if they are competent or can explain their position*)
	Giving help (general)	Help others (*Only if they are trying hard, only if they ask for help, only if I'm in a group with my friends*)
	Not wasting time/avoiding boredom	Get help from the teacher (*Because I don't want to go waste time going down the wrong track, because the teacher's way is easiest or his explanations are better*)
		Socializing (*When the math is boring*)
	Not looking incompetent (performance avoidance)	Reluctant to help others (*Because others don't see me as mathematically competent*)
		Keep people together (*Because others don't see me as mathematically competent*)
Mathematical	Press for explanations in addition to answers	Press for explanations in addition to answers (*Because I want to be convinced*)

These excerpts were assigned behavior codes *not worried about keeping people together* and *reluctant to help others* (the second quote was also assigned the condition qualifier *if they do not want to do work*). Sevanye's comments emphasize the responsibility of the individual in "knowing what they are doing," being "smart enough" to get the work done, and preparing for tests.

In contrast, Connor emphasized the responsibility of the collective, including the importance of *keeping everyone together*.

> I feel like we're all together, because we are all in a group, we are all in the same class. And we all need to get the work done by ourselves . . . everybody needs to pass math to move on to the next grade. They don't want go to summer school and stuff like that. And we have the, um, No Child Left Behind thing, and I like that. And I don't want to like get far in front of everybody else, I think everybody else should be at the same pace.
>
> I don't want to like have them behind and need to exceed past everybody. Or if I'm one of the people behind, I would like them to like, um, stay with me and help me. That's why I think it's like a group thing. Everybody should be together.

Although Connor uses the phrase, "we all need to get the work done by ourselves," the context in which this phrase occurs is much different than in Sevanye's statements. While she seems primarily concerned about her own success, and implies that everyone else should likewise be motivated, his concern is for *everyone* to be successful. Everyone needs to get the work done, and no one is to be left behind (these were given the reason code *because it helps everyone*).

When I asked Sevanye about the importance of keeping everyone together and understanding as a class, she responded:

> Yeah everyone should understand it 'cause then the next day class is gonna be like redundant because the people who were suppose to get it yesterday didn't get it. You have to go back over it and the people that know it are like, "We need to move on to the next thing." It backs the class up. Everyone doesn't learn what they're supposed to learn when they're supposed to learn it.

Although Sevanye agrees that *keeping everyone together* is important, her rationale reveals that her motivation is not connected to a sense of shared responsibility for learning. She wants everyone to understand so that time is not wasted going over material that some *individuals* already understand (reason code *if not everyone does their part it slows down individuals*). If her goal of keeping everyone together was truly a group goal, she would value the opportunity to go back over a topic because it could ensure that everyone understood, rather than seeing it as a "redundant" activity that "backs the class up."

Throughout our interviews, Sevanye expressed concern that working in a group could slow her down because of distractions and lack of knowledge on the part of her classmates. The following excerpt was assigned the codes: *Solving problems together* (*only if they are competent*), and *prefer working alone* (*because groups can slow you down*, *because groups can get distracted*).

> I don't like working in groups. Just because, like, I work at my own speed. I don't like working in groups unless everyone knows what they're doing. I don't want to leave people behind, but at the same time, when you're trying to get your work done but you can't because of talking or distraction.

This suggests that working in a group was undesirable because it could prevent Sevanye from meeting her individual goal of getting her work done.

Another contrast between Sevanye and Connor is the degree to which each expressed the goal of keeping other students accountable when working in a group. Even though Connor acknowledged that group work could be a struggle, he talked about his efforts to keep others accountable for understanding.

> If someone doesn't want to do their work in my group, usually I ask them to do some stuff, and ask them to do at least a couple of problems, help us out. So that way they are doing something, participating in some way. But if they don't want to do anything, I don't let them copy my work, and I would talk to the teacher at the end of the class. . . . If they didn't put no effort into giving me anything, or showing me how to do anything, or do anything as a group, I wouldn't let them see my answer or anything like that because they're not putting any effort into the group work.

There are a few interesting things to note in this response. First, Connor indicates that he would make some effort to get a reluctant classmate to participate (behavior code *keeping everyone together*), revealing again a concern with the success of the whole group (in contrast to Sevanye's earlier comment, "If you don't want to do your work, I'm not going to say anything to you"). Then, he indicates that if a classmate continues to refuse to contribute, then he would refuse to share his solutions (behavior code *give not just the answers but an explanation,* condition code *only if they are trying hard*). It is important to Connor that the group functions as a group, with contributions from each member. In this way, he shows a concern with keeping the members of his group accountable for making contributions and ultimately learning the material.

In contrast, Sevanye did not harbor any reluctance to share her answers with her classmates: "But if they're like, 'I don't care I'll just take the answer,' then I'll let them take the answer if that's what they want to do." In this Sevanye reveals a lack of concern with keeping other students accountable. She can *give answers* without any obligation to explain—it is up to the one who takes the answer to make sense of what it means and how it was obtained (reason code *because each person is responsible for their own understanding*).

Sevanye also implied that once she felt confident about her answers, she had no reason to listen to the ideas of her classmates: "If you already think you know the answer, when you are talking to somebody, you're not going to finish listening to what they have to say, 'cause you already think you know what you are talking about" (behavior code *don't need or want help from others,* reason code *because they can't help*). Connor, in contrast, articulated the goal of collecting the ideas of classmates, even when he knew a way of getting the answer. When I asked him to explain, he said: "Yes, it could be an advantage [for those who are advanced] because they could always learn different ways how to do it. How to do it better. And they could, um, could even do it faster than they already do it, even though they are already exceeding" (behavior code *solving problems together*, reason code *because different abilities allow us to learn from each other*).

In summary, Sevanye describes the responsibility for meeting goals primarily as falling on individuals, indicating a lack of concern with staying together as a group or keeping her classmates accountable for making contributions. Connor describes the responsibility for meeting goals as falling on the group, describing goals such as keeping everyone together and keeping his classmates accountable for making contributions.

Sevanye and Connor's viewing sessions

The contrast between individual and group goals expressed by Sevanye and Connor was evident in their group interactions. I showed Sevanye the same video I showed to Logan (described in the previous section). In the video, the students worked on the following task (Hirsch et al., 2008):

> 1. Kent County has $200,000 to spend on student salaries.
>   a. How many student workers can be hired if the county pays $2,000 per worker for a summer contract covering eight weeks? What if the county pays only $1,500 per worker? What if the county pays only $1,000 per worker?
>   b. If the pay per worker is represented by *p*, what function *h(p)* shows how the number of students who could be hired depends on the level of pay offered? (p. 360)

The following transcript comes from a short portion of the recorded episode that I watched with Sevanye during her viewing session interview.

> [*Sevanye and Lindsay talk about prom, etc. After about two and a half minutes, Sevanye reads problem 1a*]
> 1 Sevanye: Over eight weeks, so . . .
> 2 Lindsay: Here [*hands Sevanye a calculator*]
> 3 Sevanye: First, you gotta see . . . for 2000 times 8 weeks. . . . whoa, what am I doing? [*writes on paper*] What is that, 16,000? [*checks on calculator*] Yeah, 16,000.
> 4 Lindsay: Sixteen, hold on . . .

5 Sevanye: You take 200,000 and divide it by . . . [*working on calculator*] . . . 12 workers. [*Writes on paper, Lindsay alternates between writing and looking at Sevanye's paper for 18 seconds.*]
6 Sevanye: Alright.
7 Lindsay: Is it 12 workers?
8 Sevanye: Yeah. Cause you can't have a half of a person.
[*They continue to work in silence, Sevanye working on the calculator and writing. Lindsay is writing and looking at Sevanye's paper, also listening in on a nonmath conversation in another group. 55 seconds pass.*]
9 Linsday [*looking at Sevanye's paper*]: What is all this math?
10 Sevanye: You do 200,000 divided by 16,000, 'cause that's how much each worker gets paid. And that equals 12, so you can hire 12 workers. . . . Two hundred, divide it by the number, 12,000, so then 16 workers. And then you go and divide 2000 by 8000.
[*Logan joins the group. He takes about 30 seconds getting book open, etc*.]
11 Logan: What are we doing, number 1?"
[*Talk drifts off-topic. Two more minutes pass. Sevanye has been looking at her book.*]
12 Sevanye [*calling out*]: Mr. Neal!
13 Mr. Neal: Yes?
14 Sevanye: I don't understand what 2, well, 1b, is asking.

During this episode, Sevanye assumed an *expert* role in the group (Esmonde, 2009a). She explained her (incorrect) solution to Lindsay, who copied it down without challenge. However, when Sevanye herself became confused, rather than ask her classmates for help, she raised her hand and sought help from Mr. Neal. During the viewing session interview, we watched this clip and I asked her about why she went to Mr. Neal rather than talking to Lindsay and Logan.

1 Sevanye: They didn't know what they were doing. They were copying my answers.
. . .
4 I: So were you not asking them because you thought that they wouldn't be able to help, or because they didn't want to help?
5 Sevanye: Because I didn't think that they would be able to.

In this case, Sevanye's goal of resolving her confusion was for her an individual goal, one that could not be met through group processes, but through appealing to an authoritative source. Her ensuing conversation with Mr. Neal was a dialogue; while she discussed the problem with him, her group mates, Lindsay and Logan, had a side conversation. Interestingly, a parallel situation occurred later in the video, where Lindsay asked Mr. Neal a question and Sevanye in turn did not pay attention to their conversation. When I asked her about that portion of the episode, she said,

1 Sevanye: [Lindsay] just had her own question, aside from the questions that I already had.
2 I: Hmm. So the question that you ask the teacher, you see as "your" question, not as "our" question?
3 Sevanye: Yeah. I was doing, like, individual work on my own.
4 I: Right. So there might be other days, are there other days when you feel like it's more of a community question?
5 Sevanye: Yes. If you're really working together than it's always a community question, but if you're working together, but still working separately, then it's more like single individual questions
6 I: So what do you think happens most of the time?
7 Sevanye: Most of the time it's . . . mmm, it's probably more the single question from one person, more than a group question.

In this excerpt, Sevanye underscores her general emphasis on individual responsibility for meeting her goals when working in a group (turns 3, 5, 7). Although she acknowledges that there are times when the group is "really working together," most of the time they are "working together, but still working separately." These statements come in relation to an episode of group work which is characterized by low quality mathematical discourse, frequent appeals to the teacher for help, and a substantial amount of non-mathematical conversation.

Not only were Connor's expressed goals different than Sevanye's, his interaction patterns were different as well. In the following episode, he and a partner, Seth, worked on some problems in a unit on functions:

1 Seth (reading from the textbook): "Explain why light is a function of distance from light source to a receiving surface." [pause] The more intense the light then the more light . . .
2 Connor: Because it depends on how far . . . a receiving source is away from it . . . until the. . . . Light intensity, I don't know what that is. I'll have to think about it.
3 Seth: If it's closer . . . it'll be more intense . . .
4 Connor: Yeah. Okay, "Explain why the light intensity is the function of distance from light source." Um, so it would be like, um, the light intensity is a function of distance from light source . . . the farther away the receiving source is, the lower intensity it's going to be. And how close it is, the higher intensity.
5 Seth: Cool.

In this episode, both students make contributions. Although Connor is positioned as a leader, he does not take over the thinking for the dyad, and is considered an *in between* (neither expert nor novice) or *facilitator* (Esmonde, 2009c). He is willing to offer his ideas even when he is not confident about them ("I don't know what [light intensity] is. I'll have to think about it.") This provides an opportunity for Seth to make a contribution, which he does. After watching this clip, I asked Connor to talk specifically about this phrase.

> Yeah, when I was doing that, like, I didn't really understand. So, I was just kind of thinking out loud and if Seth had his opinion he could tell me like what he thinks, whether that's right or wrong. I was basically throwing ideas out there, like, "I think this is what it is but I'm not really sure" and he could help me if I was wrong [. . .] I think it's good to share your ideas and what you're thinking. You don't really have to figure it out for yourself first.

This episode and Connor's explanation reveal that solving problems is, for Connor, a task that involves contributions from multiple individuals. These contributions need not be "correct" or accompanied by confidence ("You don't really have to figure it out for yourself first.") (behavior *sharing my math thinking before knowing whether I am correct*, reason *because it helps us understand*). Connor also resisted going to Mr. Neal when he was confused; he believed he and Seth could figure the problem out on their own:

> But if we do need help we will go to Mr. Neal but we try to explain it to each other as much as we can so that we really understand. . . . We get the answer and then we explain it, that way we fully understand it. If we all understand it then we move on the next question. And then um, some groups they just go through problems just to get the work done and rush through, just so they don't have any homework. I like understanding it, I'd rather understand it than not have any homework.

Connor contrasts his goals for understanding with "some groups," who merely want to finish the problems. Furthermore, he expresses the importance of understanding as a group, by explaining so that "we fully understand it" (behavior *solving problems together*, reason *because it helps us understand ideas better*).

Group and Individual Goals

The cases of Sevanye and Connor show a contrast between a student who seems to view responsibility for meeting goals as lying almost exclusively with the individual and a student who seems to view responsibility for meeting goals primarily as being shared across members of a group. In contrast to the interaction observed in Sevanye's group, in Connor's group, contributions were made by both Connor and Seth, neither was positioned as an expert, and opportunities were provided for collaborative revision of ideas.

The contrast between Connor and Sevanye can be seen in Tables 2 and 3. Table 2 shows a consolidated list of Sevanye's goals, along with the associated behaviors and qualifiers. Table 3 shows a similar list for Connor.

The individual/group dimension of students' goals can be juxtaposed with the mathematical/nonmathematical dimension. Figure 3 shows these two dimensions forming four quadrants, each representing a combination of goals types from each dimension. For teachers

TABLE 2
Sevanye's Goals

Goal Type	Goal	Behavior *(Qualifiers)*
Individual	Getting explanations or answers (from peers or teacher, no evaluation)	Get help from the teacher *(Only if I am especially confused about the problem)*
		Get help from peers (*Only if they are competent*)
		Get explanations in addition to answers (*Because it helps me understand ideas better*)
	Giving answers or explanations (no obligation to check understanding)	Give answers only (*Because each person is responsible for their own understanding*)
	Getting through tasks individually	Not concerned about keeping people together (*Because each person is responsible for their own understanding*)
		Keep everyone together (*If not everyone does their part, it slows down individuals*)
		Reluctant to help others (*Because each person is responsible for their own understanding*)
	Not wasting time/avoiding boredom	Helping others not important (*if they do not want to do work*)
		Get help from the teacher (*Because I don't want to waste time going down the wrong track, because the teacher's way is easiest or his explanations are better*)
		Solving problems together (*Only if they are competent*)
	Mastering material in isolation	Prefer working alone (*because groups can slow you down, because groups can get distracted*)
		Don't need or want help from others (*Because they can't help*)
Group	Solving problems together; resolving disagreements	Resolving disagreements (*Because it helps you understand ideas better, because you get to see it multiple ways*)

TABLE 3
Connor's Goals

Goal Type	Goal	Behavior *(Qualifiers)*
Individual	Getting explanations or answers (from peers or teacher, no evaluation)	Get help from the teacher *(Because it helps me understand, because I want to know what not to do)*
Group	Wanting to be convinced (pressing peers for explanations, evaluating explanations)	Press for explanation in addition to answer (*Because I want to be convinced*)
	Keeping other students accountable	Give not just answer, but explanation (*Only if they are trying hard*
	Staying together, making sure everyone understands	Get help from the teacher (*Only if we are especially confused about the answer*)
		Keep everyone together (*Because it helps everyone*)
	Solving problems together; resolving disagreements	Solving problems together (*Because it helps us understand ideas better, because it helps everyone*)
	Collecting ideas even if a solution has already been found	Solving problems together (*Because different abilities allow us to learn from each other*)
		Sharing my math thinking before knowing whether I am correct (*Because it helps us understand*)

who want to use group work, goals like those in the upper right quadrant are likely to be most compatible with their efforts. However, one striking feature of this representation is the extent to which Logan and Sevanye's goals fall in the other three quadrants. This diversity presents significant challenges for teachers seeking to use group work. The case of Sarah, described in the next section, illuminates another such challenge.

Normative Goals at Conflict with Personal Goals: The Case of Sarah

While the mathematical/nonmathematical dimension has to do with *what* students are trying to achieve, and the individual/group dimension has to do with *who is responsible* for meeting goals, the third dimension has to do with the extent to which described goals are perceived by students as being imposed on them by external sources rather than goals with which they personally identify. In other words, this dimension attends to the question, *whose goal is this?* This dimension was more difficult to detect and categorize than the previous dimensions, particularly because students may express a goal without clarifying the extent to which they personally identify with it. The case of Sarah provides an example where personal goals seemed to clash with perceived normative goals.

Sarah's goals

Sarah presented a unique case in that unlike her classmates, she was unabashed in her negative sentiments about working together in Mr. Neal's class. Her expressed behaviors were overwhelmingly antigroup: she was not concerned about keeping the group together, did not want help from others, was reluctant to help others, and generally preferred to work alone. However, there were a

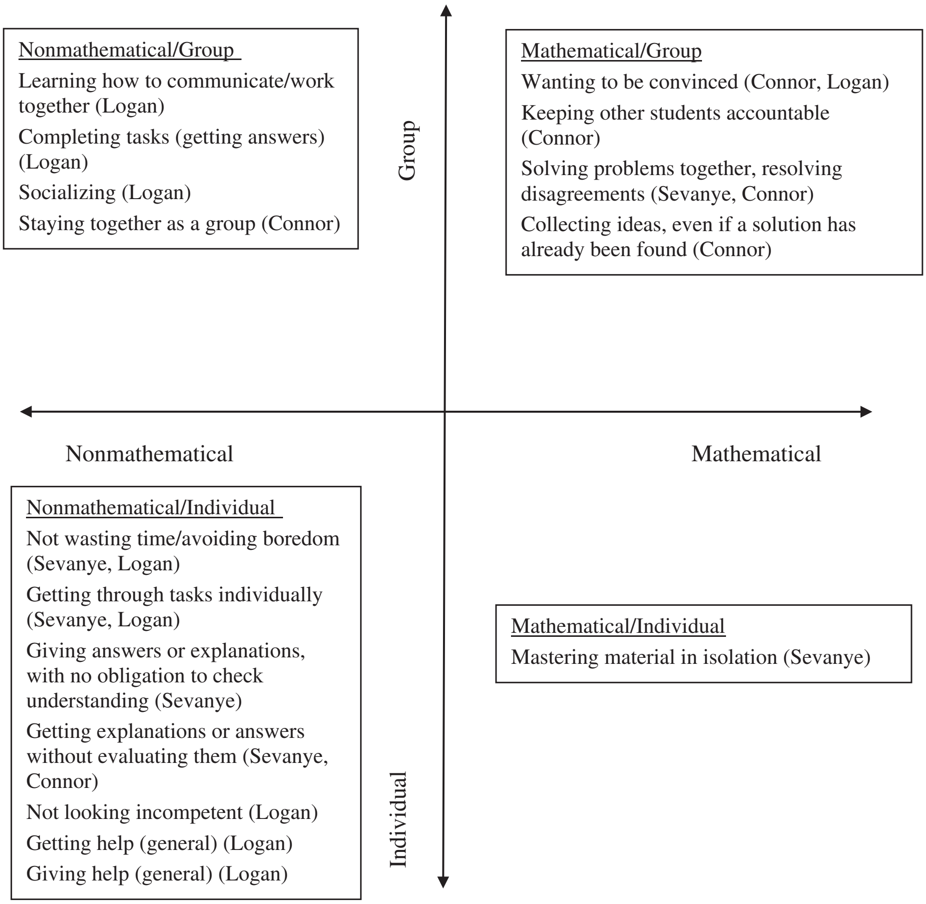

FIGURE 3 The mathematical/nonmathematical and individual/group dimensions of students' goals for working together.

few moments during the interview in which Sarah made statements about the purpose and effect of group work that seemed to contradict these sentiments. For instance, Sarah appeared to indicate that communication was important and that, under certain conditions, she could appreciate working in a group. A detailed investigation of the data led me to question the extent to which Sarah actually affiliated with these goals, prompting the conjecture that she was merely voicing goals that she perceived to be embedded in classroom expectations.

For example, when I asked Sarah during our first interview, "What do you think is the purpose of group work?", she replied: "Like we all get together and see if the answer is right or what's wrong, and how to communicate with each other." Then she followed this by saying, "I understand the communication but it's lame." When I asked her to elaborate, she replied, "I understand how we're supposed to communicate in this world." Sarah reiterated this later, saying "they're trying to implicate [*sic*] a real work situation." The use of "they're" and her characterization of the goal as "lame" suggest that this goal of learning to communicate in a "real work" situation is not actually a purpose for working together with which Sarah identifies. It is a goal established by someone else.

But what does Sarah mean when she describes this goal of learning how to communicate? One interpretation is that she sees working together as a strategy that teachers use at least partially to help student learn to work together, because people in "real work situations" have to work together. That is, even if communication does not help one learn mathematics, communicating is a goal in itself. (I do not claim that this view was actually endorsed by Mr. Neal; I only suggest that Sarah *perceived* this to be a goal of group work that was embedded in class expectations. Logan also expressed this idea about group work.)

It was evident throughout Sarah's interview that "learning to communicate" was a not a personal goal for her. "I work independently. I work actually perfectly fine by myself, it's just people, like, they're annoying. And they talk and they're not doing work, and they never finish." For Sarah, attempting to work with her classmates was difficult because she perceived them as distracting and off-task. The problem was not communication *per se*, it was that when she tried to work with her classmates, they did not communicate in a way that she saw as productive. She made several statements to this effect, including "they just go around in circles," (coded as *because they can't help*) "people just [keep] bringing up random things," "they won't pay attention" (coded as *because the group can get distracted*), and "I end up doing the work, well, most of it" (coded as *because there is an unequal distribution of work*). What Sarah appears to dislike about working in groups is the nonmathematical aspects of communication—exactly the thing that Sarah described as "lame." "Learning to communicate" as its own goal holds little value for Sarah. She told me that she eventually abandoned working together: "I gave up about a few weeks ago. I'm like, yeah, I wanna keep to myself in my corner now."

The interpretation that Sarah perceived an external goal about learning to communicate comes across most clearly when Sarah acknowledges that working together could *potentially* achieve the goal of "figuring out the problem," but only under certain conditions.

1 I: So the reason why you think [Mr. Neal uses group work] is to help people communicate with each other?
2 Sarah: Yeah. And to figure out the problem. And what they can't understand or not understand.
3 I: So, do you think that works?
4 Sarah: At times it does, when they actually focus.
5 I: Can you tell me more about that?
6 Sarah: When they shut up and start getting to work. . . . When they actually do the work and then they're like, "ok what are we gonna do now?" Yeah. That's when they do it.

At several points during the interview, Sarah emphasized that group work of the quality she describes here were rare occurrences in Mr. Neal's class. This excerpt can be interpreted as a description of an imagined scenario in which Sarah's desire to learn mathematics could be achieved through interactions with her group mates. In this scenario, her group mates are not learning to communicate as an end in itself; they are communicating with a mathematical purpose. She explicitly refers to "focusing" (turn 4), and "shutting up" and "actually doing the work" (turn 6). These behaviors are all about not getting sidetracked by nonmathematical conversations. For Sarah, discussing mathematics is a goal that is in conflict with the perceived normative goal of learning to communicate as its own end. Only in cases where nonmathematical communication is dropped from the normative agenda, where focusing on the mathematics is the primary goal of the group, can working together come into alignment with her personal goal of learning mathematics.

Normative versus personal goals

After examining Sarah's case, I determined that a third dimension of students' goals for working together needed to be created. This dimension would differentiate between goals that students perceive, but do not necessarily share, from goals that students adopt for themselves. I call the former normative goals and the latter personal goals. This use of the word "normative" is not meant to imply that the goal is actually shared by the members of the community, but rather that the student *perceives* the goal to be normative; that is, they perceive an expectation on the part of a teacher or others that this is something that they *should* be trying to achieve. Any of the previously mentioned goals could fall differently along the normative/personal continuum (see Figure 4).

Normative goals are most easily recognized when students voice an expected behavior along with some degree of resistance to the behavior, as Sarah did with the goal of learning to communicate as its own end. However, normative goals are probably the most difficult to detect, as

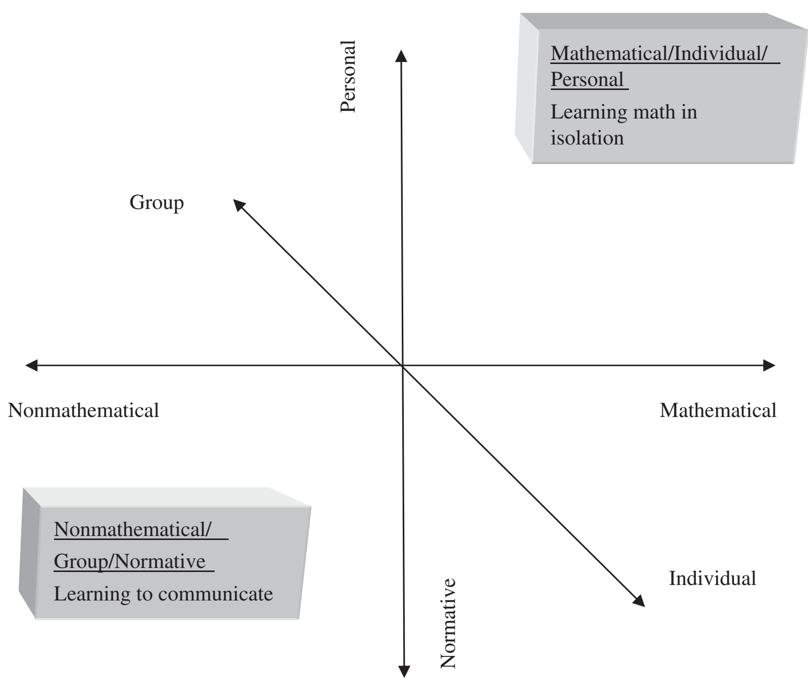

FIGURE 4 The three dimensions of students' goals for working together with Sarah's conflicting goals.

many students may, in an interview, parrot the normative goals espoused by the teacher even if they do not adopt them for themselves (Kotsopoulos, 2010). In my analysis, a key signifier of a normative goal was the stark contrast in the behaviors that Sarah's described in her interviews.

DISCUSSION

Practical Rationality and Students' Goals for Working Together

The practical rationality framework described by Aaron (2011) posits that there are shared social resources that guide decisions made by students. These resources are mediated by students' personal resources, including their goals. This study teases out the role of students' goals by defining three goal dimensions that are related to the practice of working together.

In the following sections I describe how each dimension is related to previous research and connect the findings to the practical rationality framework by describing how students' goals can be seen as mediating norms and obligations. I also provide some suggestion for using students' goals as leverage for prioritizing obligations to encourage more productive behaviors.

Mathematical and social goals

In some studies, students' mathematical goals and social goals have been described as working in complementary fashion (Hatano, 1988; Jansen, 2006; Walter & Hart, 2009). However, in Logan's case, his social goals worked against mathematical collaboration. He did not see communicating with his peers as helpful for learning mathematics, yet enjoyed group work because it gave him opportunities to meet his social goals (Clarke & Xu, 2008; Williams & Baxter, 1996). This dimension of student's goals is important for the practical rationality framework because if student decisions are driven mostly by social goals, this may put emphasis on some obligations at the expense of others (see Figure 1). In particular, students like Logan may be less influenced by obligations to truth, the institution of school, or concern for interpersonal relationships in the class generally, but instead may respond primarily to obligations stemming from their personal relationships with other individuals. They may seem to comply with classroom norms that involve engaging in discussions, but may do so only when they work with their friends, or, as in Logan's case, may even conflate "working well" with "enjoying talking."

For teachers, understanding these goals and how they influence studenting has important implications. Could Mr. Neal have used Logan's obligations to his friends as leverage for getting him to talk about *mathematics*? For example, Jansen (2006) suggested that some students who would normally not engage in a discussion might do so if they believe their explanations can help others. In the present case, if Logan could be convinced that he had something to offer his classmates (and particularly, his friends) mathematically, he might see engaging in mathematical discussions as a way of earning status with his friends and thus achieving his social goals. Rather than avoiding putting Logan in a group with his friends, a teacher might put Logan in a situation where his friends depend on his mathematical help in order to meet their own goals. This strategy takes into account Logan's filtering of obligations and ties his primary obligation (individual relationships) to the teacher's goal of improving collaborative behavior.

Group goals and individual goals

The present study adds to research on the locus of responsibility for meeting goals by including the voices of Connor and Sevanye, two classmates who locate this responsibility very differently. Combined with the mathematical/nonmathematical dimension, we begin to see a broadened space within which goals may fall (see Figure 3). Unlike Logan, Connor and Sevanye both have mathematical goals. But for Connor, the locus of responsibility for meeting those goals lies with the group, while Sevanye sees individuals as being responsible for meeting these goals.

As with Logan, we can see that these different kinds of goals filter the social resources of practical rationality differently. For both Connor and Sevanye, obligations to truth appear to be important (both are concerned with understanding mathematical ideas), but for Connor, obligations to interpersonal relationships take on much more significance than for Sevanye. Sevanye generally does not look to her classmates for help, instead seeking help from Mr. Neal, while Connor sees contributions of his classmates as valuable and also senses an obligation to make sure others understand. In general, Connor seems to possess a favorable disposition towards collaboration, while Sevanye does not.

A teacher could potentially use Sevanye's obligation to truth as leverage for improving her engagement in discussions, provided that he could help her think about the purpose of group work differently. Part of what keeps Sevanye from interacting with her classmates is that she seems to see the primary purpose of group work as helping or getting help (described in a previous paper, see Webel, 2010a). Because she sees herself as mathematically competent, others cannot provide help, so group interactions serve little purpose for meeting her mathematical goals. Rather than attempting to develop in Sevanye an obligation to help others, it may be more productive to help her see learning mathematics as a process that is less about giving or receiving information and more about refining one's current ideas. With this view, mathematical discussions might be take on more utility for her because her role would not be limited to merely helping others, but would include "building" knowledge together (Mueller, Maher, & Powell, 2007; Scardamalia & Bereiter, 2006).

This may be accomplished by limiting the amount of mathematical scaffolding provided by the teacher, which models asymmetric "help giving" (Esmonde, 2009a; Webb et al., 2006), and instead providing social scaffolding, which emphasizes symmetric interactions between students (Dekker & Elshout-Mohr, 2004). This could be accompanied by frequent breaks in small group work for the class to come together and discuss ideas in order to prevent frustration with lack of progress (Webel, 2010b) (such breaks did not occur in the recording I watched with Sevanye). Through such practices, students like Sevanye might come to see group processes as *supporting* her obligation to truth rather than detracting from it. More work is needed to explore whether such interventions might be effective for students like Sevanye.

Personal and normative goals

The distinction I draw between personal and normative goals is similar to that described by Cobb and colleagues (2009) in defining personal and normative identities. According to these authors, normative identities are defined by the classroom obligations with which students would have to identify in order to "develop an affiliation with classroom mathematical activity and thus with the role of an effective doer of mathematics, as they are constituted in the classroom" (p. 46). Personal identities acknowledge that students may to varying degrees identify with, comply with, or resist these normative obligations. Likewise, I use normative goals to describe those goals that are perceived (by individuals) as being established as normative in their class, and personal goals are those goals that students adopt for themselves, which may be in conflict with normative goals.

Recognizing that goals can vary from normative to personal emphasizes the negotiated nature of students' goals. It is likely that most goals for students sit somewhere between the goals that they bring to the classroom and those which are endorsed and embodied in a particular mathematics class (see Figure 4). Sarah's interviews reveal a large gap between her personal goals and the perceived normative goal of learning how to communicate, and Sarah struggled to negotiate this dissonance.

There may be other conflicts between personal and normative goals. For instance, a normative goal might be to use mathematical arguments to come to a conclusion, but a personal goal might be to avoid arguing in the group. Exploring these kinds of conflicts is worthwhile task for future research. One recommendation for teaching is that teachers be much more explicit about their reasons for asking students to work together (Adler, 1999; Pimm, 1987), which may help students connect their personal goals to the normative goals the teacher intends to endorse.

From the standpoint of practical rationality, we see how Sarah's personal goals, which emphasized obligations to truth, seemed to her in direct conflict with a perceived norm that emphasized interpersonal obligations exclusively. The interpersonal obligation was filtered out to the extent that Sarah virtually abandoned group work. One implication of this study is that the negotiation between personal resources (including goals) and the goals endorsed and promoted by the teacher needs further study. This suggests a blending of research methodologies which, on the one hand, tend to focus exclusively on normative activity in the classroom, and on the other, focus on individuals' perspectives in a classroom setting (Cobb & Bowers, 1999; Cobb et al., 2009). Research along these lines could explore how students come to identify with or reject normative goals and the factors that affect this process.

Diversity of goals

In this study, one striking feature is the variety of different kinds of goals for working together that existed within Mr. Neal's class. Of course, we only have the perspectives of a few students, who were selected because they represented divergent views about working together. However, Mr. Neal's class is itself a case of a mathematics class in which a teacher wants to promote collaboration through the use of small groups. Understanding the individual cases represented by each of the students helps us see why this can be a challenging prospect. Students like Sevanye are mathematically focused and competent, and yet these assets are not put to use in group work because she sees understanding as an individual responsibility. Logan enjoys group work, but does not leverage these opportunities towards mathematical ends. And Sarah rejects group work completely, seeing it as prioritizing a goal that she does not share. Supporting group work means attending to these individual goals and providing a structure in which these goals are more likely to work toward the purposes envisioned by the teacher.

CONCLUSION

In this article I explored students' goals for working together in a class that was centered on solving problems collaboratively. I investigated their goals not only as expressed in a general interview or through a survey, but tied to context through viewing session interviews. A key contribution of the study is the development of the three dimensions of students' goals for working together in mathematics classes. These dimensions are more targeted than those studied in traditional achievement goal orientation literature (Ames, 1992; Pintrich et al., 2003), and attend to different features of the goal, such as the locus of responsibility for meeting the goal. This article also reveals a tremendous amount of variation along these dimensions among the students in a single class, which emphasizes the need for research which acknowledges the diverse goals that students bring to the classroom. Finally, this article links the goal dimensions to a larger framework which acknowledges that both individual resources like goals and social resources like norms and obligations come into play in a mathematics classroom. Future work should continue to explore how students negotiate these two kinds of resources as they make decisions and justify their actions in the mathematics classroom.

Limitations

There are a number of limitations to this exploratory study. It is a very small sample, and there is a need for more empirical validation of the dimensions identified. Also, the identification of perceived normative goals might have been enhanced by analysis of the norms as enacted in Mr. Neal's class. Not enough observational data of all students was collected to support such an analysis. This study was also limited in that it relied upon interpretation of students' expressed goals, although this interpretation was aided by the viewing session interviews. These helped students ground their descriptions of their goals in the context of their studenting practices, at times prompting them to add qualifiers that aided in interpreting their goals.

Future Research

In identifying multiple dimensions upon which students goals for working together can vary, this study raises questions for future research. For example, how do various combinations of these goals play out in the mathematics classroom? What are the implications of putting students with primarily individual, mathematical goals in a group with students who have nonmathematical, group goals? How malleable are these goals? How are their goals influenced by classroom norms? What factors influence the process of students coming to identify with normative goals? What factors inhibit this process? Also, now that the dimensions have been identified, future research might pursue ways to measure these goals through less intensive means, such as surveys or interviews where students watch animations or videos of other students rather than videos of themselves.

ACKNOWLEDGEMENTS

I would like to thank Amanda Jansen and Wendy Aaron for their feedback on earlier versions of this article.

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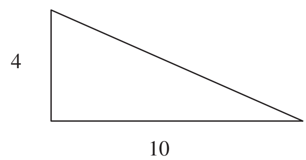
APPENDIX

Student Interview 1 Protocol (Sevanye)

1. On the survey, you said you neither agreed nor disagreed with the statement that solving problems is mostly a job for students to do in groups (item 27). So I am curious; what do you think about working in groups?
    a. Do you like working in groups? What do you like about it? What do you dislike about working in a group?
    b. What do you think is the purpose of group work? Does it work?
2. I know you switch groups fairly often. Do you feel like there is a kind of "base group" that you are with most often?
    a. If so, who are the other people in your group? What do each of them do when you are working on mathematics (how do they help?) What do you do, what is your job?
    b. If not, how would you describe the jobs that different people in a group have? Does everybody do pretty much the same thing, or do some people do different things? What do other people do, and what do you do?
3. If you didn't come to school one day, how do you think your group would be different?
4. Tell me about a time when someone in your class helped you with a math problem.
    a. Did they have to convince you that their answer is right? If so, how did they?
    b. Why did you want them to explain?

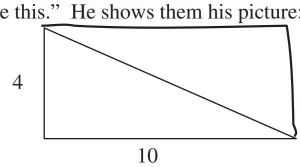
A group of students is working on a problem where they are trying to find the area of this right triangle.



Jane, Joe and Frank are working in a group. They each get an answer, and then decide to share with each other. Frank explains what he did to solve the problem: "I multiplied base times height. Four times ten is forty."

Jane replies: "No, you forgot a step. See, it's just one-half base times height. See, look at what I did. I just took the base, 10, multiplied by ½, and I got 5. Then I multiplied by the height, 4. So I knew the answer was 20."

Joe, after listening to Jane and Frank, says "I did it a different way. I drew another triangle, just the same, but on top, like this." He shows them his picture:



"Now I have a rectangle that is 4 by 10, so the area is 40, just like you said, Frank. But since each triangle is only half of the rectangle, its area must be half of 40, so I knew it was 20."

FIGURE 5 Vignette used in Interview 1.

5. On survey item 20, you gave a 3 for "when I think I have a right answer, I usually try to convince my classmates that it is right." So what usually happens when you have an answer that you think is right?
   a. What if someone else is confused about your answer? What do you do?
   b. What if they still don't understand?
6. [Refer to survey item 9: A disagreement with my classmates sometimes helps me understand math better]. Can you talk a little more about this? Can you think of an example of a disagreement you had in a group and how that helped (or didn't help) you understand something?
7. I want to understand a little bit more about what you mean when you talk about helping others. Here's an example of three students working in a group. [Interviewer reads vignette; see Figure 5]
8. What do you think about the help provided by Jane and Joe? Which explanation do you prefer, and why? Which one do you think would be most helpful for Frank? Why?
9. I've noticed that sometimes Mr. Neal leads a discussion before you start an investigation, and sometimes he does so after you've worked on some problems. For example, the other day you were talking about the Think About This Situation (TATS) on page 196. I noticed

you participated a few times during the discussion. Do you feel like you were engaged during this discussion?
   a. Are there times when you feel like you can't or don't want to participate?
   b. Do you think you would participate in the same way in another class with another teacher?
10. I've noticed that during these discussions, Mr. Neal sometimes doesn't tell the class whether or not someone's answer is correct. Instead, he asks people to decide whether or not they agree with the person's solution. How do you feel about that? What do you think is the purpose of this strategy? Do you think it is working? What could the teacher do to make it work better?
11. Do you feel like you participate differently in this math class than in other math classes? If so, how, and why?
12. [IF TIME PERMITS] Is learning math different than learning English or history? Why?
   a. How do you know whether or not your answer is right in math?
   b. Some people might say that the only way they can know whether or not their answer is right in math is by asking the teacher. Would you agree with that? Why or why not?

Student Interview 2 Protocol (Sevanye)
Show video clip, part 1 (Sevanye, Lindsay, and Logan, Kent County Task):

1. Any thoughts about this clip?
2. During this clip there are a few times where you getting help from Mr. Neal. It seemed like you kind of went straight to Mr. Neal rather than asking Logan and Lindsay what they think. Why is that?
3. Lindsay and Logan don't seem to be paying much attention when you are talking to Mr. Neal. I asked them why . . . what do you think? Does that concern you at all? Do you think they are getting it?

Show next section of clip:

1. Lindsay says what she thinks the equation means. Did you understand what she was saying?

Show final segment:

2. There does not seem to be much math talk . . . people are looking at their books, but when you talk it is about other things, not the math. Why is that?
   a. If it is that you don't care, then why do you not care in this particular case?
   b. Are there situations in which you care more? Why?
3. Do you feel like you learned anything during this session? What? Do you think in some sessions you learn more than this one? Or is this about right for group work? Why?