HIGH SCHOOL STUDENTS' PERPSECTIVES ON COLLABORATION IN THEIR MATHEMATICS CLASS

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Collaborative learning formats are being used increasingly in mathematics classrooms. However, students working in groups often remain dependent on their teacher as the distributor of knowledge, or turn to peers to fill this role. In this project, I used viewing session interviews to investigate high school students' perspectives regarding collaboration in their mathematics class. I highlight two students who characterized their responsibilities as class members differently, but expressed similar beliefs about the purpose of group work and the nature of mathematical understanding. I argue that viewing collaboration as "helping" may limit the potential benefits of having students work together.

Introduction

Collaboration and Identity

Students' experiences in mathematics class, including their experiences with collaborative learning formats, can affect their identities as learners and doers of mathematics (Boaler, 2002). Cobb, Gresalfi, and Hodge (2009) used interviews to show how students in two classes with very different pedagogical approaches constructed identities differently in terms of their mathematical obligations to each other, their opportunities to exhibit agency, and their view of the teacher as a mathematical authority. But even in a class where student interaction is a central component of instruction and where the teacher seeks to establish a community approach to learning, students may construct mathematical identities that are individualistic and passive in nature. These identities are influenced not only by the nature of instruction, but also by their beliefs about the purpose of collaboration in their mathematics class, as well as their beliefs about mathematical understanding more generally. These beliefs may be resistant to change, and may constrain the ways that students are able to engage in collaborative work.

In this paper, I compare and contrast the perspectives of two high school students in the same mathematics class where collaborative work was common. Sevanye (a pseudonym) described her responsibilities primarily in individualistic terms, rarely expressing concern for contributing to the understanding of her peers. In contrast, Jordan's description of learning was more community-focused, acknowledging a responsibility not only to improve her own knowledge, but to help improve others' knowledge as well. At the same time, both students seemed to hold similar beliefs about the purpose of collaboration and also about the nature of mathematical understanding. Both Sevanye and Jordan described working together primarily as passing knowledge from capable students to struggling peers, and seemed to see mathematical understanding as knowing procedures. I argue that these beliefs may be related to each other and that they have implications for the strategies that teachers use to implement collaborative learning in their classrooms.

Framework and Research Questions

Particularly when they are working on problems for which solutions are not already known, students will inevitably struggle. Clearly, teachers in classrooms with 20 or 30 students will not be able to attend to each student and help them resolve their confusion. Putting students into groups can be seen as one way of resolving this dilemma, as the stronger students can help those who are struggling. Some research has suggested that students working in groups tend to position themselves asymmetrically, where select students take on an "expert" role, telling other students what to do, while their peers defer to their authority (Esmonde, 2009; Kotsopoulos, 2007).

Some strategies for implementing collaboration seem to assume (and may even promote) asymmetric positioning by, for example, training students to give better explanations when helping their peers (Fuchs et al., 1997; Webb & Farivar, 1994). Even in a study which claimed that students created a "collaborative zone of proximal development" in their classroom, descriptions of student interaction suggested asymmetric positioning: "Some students additionally adopted teacher-like scaffolding strategies to assist less capable peers, for example, by asking questions that led their partner to locate an error or reconsider a plan" (Goos, 2004, p. 282). While these interactions deemphasized the *teacher* as the distributor of knowledge, some students still seemed to be positioned as knowledgeable while others were positioned as less able to contribute.

Another view of collaboration suggests that students could be positioned more symmetrically while generating knowledge. According to this view, knowledge need not be possessed by any individual in order for it to emerge among a group of learners (Davis & Simmt, 2003). That is, students may *co-construct* new ideas through activities such as reiterating, redefining, or expanding on the ideas of others (Mueller, Maher, & Powell, 2007). Scardamalia and Bereiter (2006) argue that this is precisely what takes place in research institutions—groups of people come together, construct new ideas through discourse, and together advance the knowledge of society. This process of *knowledge building* should not be limited to the research laboratory, these researchers argue; it should be integral to schooling.

One of the goals of this paper is to argue that the former view (collaboration as helping) may undermine the benefits that might be gained from having students work together, particularly if this view is adopted by students and teachers. I set up this argument by using multiple lenses to examine two students' perspectives on working together. First, I investigate the extent to which each student describes personal obligations to help her peers, as well as the extent to which each describes her classmates as resources for helping her. Then I examine each student's beliefs about the purpose of collaboration; in particular, does she describe working together as a process of building new knowledge through the contributions of multiple individuals, or as a process of passing ideas from knowledge-giver to knowledge-receiver? Finally, I explore each student's beliefs about the nature of mathematical understanding in order to explore connections between these beliefs and beliefs about the purpose of group work.

Participants and Data Collection

Sevanye and Jordan were both in Mr. Neal's "Integrated Math 3" class, the third in a sequence of courses taught with the Core Plus mathematics curriculum materials (Hirsch et al., 2008). They were both in 10^{th} grade and both reported typically receiving B's in mathematics.

I spent 23 days in Mr. Neal's class, recording video of whole class discussions and small group work. I interviewed eight students from the class twice, including Sevanye and Jordan. In the first interview, I asked them questions about working with their peers, such as whether they liked working together and why, what they saw as the goal of group work, what they saw as their role or "job" when working in a group, and under what circumstances they felt comfortable sharing their ideas publicly in whole-class discussions. In the second interview, I showed each student video clips of their own participation in various collaborative activities and asked them questions about how they participated. Using these viewing sessions helped students be more specific about their roles, and allowed them to address any inconsistencies between stated beliefs and observed behavior. The data was analyzed by categorizing portions of interviews according to recurring themes that were developed and refined throughout the analysis process (Glaser & Strauss, 1967). In this paper, I focus on three of these themes: how Sevanye and Jordan characterized their personal responsibilities as members of the class, how they described the purpose of collaboration, and how they talked about mathematical understanding.

Results

Responsibilities: Individual Versus Community

When Sevanye discussed her role during group work, she emphasized a responsibility for making sure *she* understood rather than a responsibility to the members of her group for improving collective understanding. During the initial interview, I asked her about situations where her group mates might get off-task, and she shrugged, saying,

> If you don't want to do your work, I'm not going to say anything to you. We're all in high school, graduating in two years. If you can't be smart enough to know I need to get this work done 'cause there's a test next week then…that's not my problem.

In another section of the interview, she said, "It's up to everyone…. I can't expect anybody else to carry me 'cause I don't know what I'm doing. That's not fair to anybody. Especially if they know what they're doing 'cause they paid attention." These comments suggest a rejection of obligations to contribute to her peers' understanding.

In one video clip, Sevanye was seen allowing a group mate to copy her answers. She explained the behavior in the interview: "It doesn't bother me, as long as in the end she understood what we were doing. If you wanna copy my answers, I mean…I don't always get the right answers, so if I mess up I'm gonna have to erase." Here Sevanye does mention her classmate's understanding, but since she made no effort to help her peer understand, this can be interpreted as a responsibility she put on her classmate, not herself. In general, for Sevanye, the responsibility for understanding seemed to lie with the individual rather than the community.

Unlike Sevanye, Jordan did express more of a concern for helping others understand. She said, "…we're supposed to help each other or whatever, and I don't know, I just don't feel as though we should go on, leaving one person sitting there stuck, because…I don't want to be left there stuck." Jordan seemed to see helping others in terms of reciprocation—helping others was important because she might need help from them at a later time. This came up when discussing the nature of help as well: "Like, I don't like giving people the answers, like I like explaining it to them, because I don't want nobody giving me the answer. Like I like if somebody explains it to me so I can know how to do it." Here Jordan seems to be expressing a responsibility not only for sharing her answers, but for explaining how to solve problems.

Sevanye and Jordan also differed in the extent to which they saw their peers as able to help them understand. In many cases Sevanye described her classmates as incapable of helping her. In one episode where she became confused, rather than ask her group members about their ideas, she raised her hand to call the teacher over for help. When I asked her about this, she replied,

> They didn't know what they were doing…. I asked Lindsay a little bit about like what she was doing, and she didn't really seem to know. If I would have asked her some more, she probably would have confused me more than what I was already confused about…so I just went right to Mr. Neal.

In this case, Sevanye did not seem to see her peers as resources for helping her understand.

Sevanye also said, “When I work in group I like to work with people who already know what they’re doing,” but other comments suggest that this was a rare occurrence for her. In general, Sevanye seemed to prefer explanations from Mr. Neal rather than from her classmates. In one episode, Mr. Neal conducted a whole class discussion in which he gave an explanation for a solution that a student had written on the board. I asked Sevanye if she would have preferred that the student explain his solution rather than Mr. Neal. She replied, “No. I like it when Mr. Neal explains it better. Like, he just breaks everything down…. I’d rather Mr. Neal do it because he knows exactly what he’s doing.” This quote again suggests that Sevanye did not see her peers as able to help her understand, and seemed to see Mr. Neal as the primary mathematical authority in the classroom.

In contrast, Jordan explicitly described her peers as capable of helping her learn. She stressed the importance of sharing ideas with others:

> Yeah I like group work because you get to share your ideas, and everybody has different thoughts or whatever. I think it’s better to see what everybody else thinks about the problems or whatever, because….like Mr. Neal says, more approaches to a problem, whatever…so we don’t have to just do it the way he shows us.

Jordan acknowledged that it was possible to learn from others without relying on Mr. Neal. In the case of a disagreement between students, she said that “if you both explain how you got your answer, then maybe you all can come up with something. Maybe if you go through it again, you see somebody make a mistake, something like that.” Jordan acknowledged that “some students come up with their own ways, like ways that the teacher doesn’t even explain yet,” although she claimed that this was rarely the case for her.

Jordan indicated that she valued other students’ explanations during whole class discussions as well as during group work. She said that rather than Mr. Neal providing all of the explanations, he should allow students to explain because “maybe the student had a different approach than the teacher had. Or maybe some students understand it better from a student.” Jordan seemed to see students as capable of contributing to each others’ understanding, and wanted Mr. Neal to provide opportunities for students to share their ideas with their peers.

The preceding sections have outlined some ways that Sevanye and Jordan described their roles in Mr. Neal’s mathematics class. While Sevanye primarily described her responsibilities in individualistic terms, Jordan expressed a sense of responsibility for helping others and also saw her peers as resources for improving her own understanding.

Beliefs About Collaboration

While Sevanye and Jordan’s expressed obligations as members of their mathematics class seemed different, they described similar beliefs about the purpose of collaboration. In particular, both Sevanye and Jordan expressed the view that working together was primarily about those with knowledge “giving” that knowledge to those without. In other words, they described collaborative work in asymmetric terms, with some students positioned as knowledge-givers and others as knowledge-receivers. Sevanye was quite explicit about this. When asked about the purpose of group work, she said, “You put the stronger people with some people who may not know the subject as well and it helps them.” She also indicated that “being knowledgeable ” was a prerequisite for being able to help others. “I wouldn’t want to tell somebody how to get the answer if I wasn’t one hundred percent sure how to do it,” she said. As I described in the previous section, she also was hesitant to listen to her classmates if they did not have a solid understanding. This suggests that for Sevanye, working together was primarily about giving knowledge to others who were confused. This activity seemed to hold little value for her, which predictably resulted in a generally negative view of group work.

Jordan also seemed to believe that working together that was primarily about students "helping" each other. When I showed her a video clip where there was very little discourse between her and her group members, she explained: "When somebody needs help, we help each other.… Even though we work at a different pace, we still stop if you don't understand something, we'll explain it." Here Jordan is again expressing a view that learning is a collective effort, but she is also describing this collective activity in terms of *stopping* and *helping*, particularly when someone *does not understand*. This implies that some members of the group already do understand and could go on if they were not obligated to help their classmates. It also suggests that for Jordan, a primary reason for discourse is to express confusion and get or give help. Later she talked specifically about a partner who helped her understand. "I know my partner, it makes sense to him, 'cause he always knows. I'm like, 'do you understand this?' And he's like, 'yeah.'" She described a specific case where she was confused: "…so I asked my partner. He understood it. He explained it to me." In this sequence she describes her partner as knowledgeable, herself as lacking knowledge, and working together as the process of him explaining to her. These quotes suggest that for Jordan, working together was often a process where students who had good understanding explained to students who did not.

So while Jordan described her responsibilities in the classroom as extending beyond improving her own understanding, both she and Sevanye described working together in terms of passing knowledge between students. This finding suggests that promoting collaboration in the classroom may mean more than getting students to feel obligated to help each other. It may also involve addressing students' beliefs about the purpose of group work, beliefs which may be tied to students' views about the nature of mathematical understanding.

Beliefs About Mathematical Understanding

During the interviews, Jordan and Sevanye articulated the belief that mathematical understanding consists primarily of knowing procedures. This was apparent in their descriptions of what they valued when giving and receiving help, which often included concerns with *what to do* rather than providing and evaluating arguments for *why* ideas were true.

When I asked Sevanye about how working in a group might help one understand, she said, "Because whenever you do something wrong you know how not to do it…and how you should do it." Her emphasis on *how to do it* suggests a procedural focus, and this was repeated throughout both interviews. Here she explains an episode where she had the wrong solution:

> I was in a group…and we had both gotten the answer but my answer was wrong. And I asked [my partner] how she got it. She had, her answer was right. I didn't put a negative sign where it was supposed to go and she was like, 'oh, well this is how you're supposed to do it. But you just have to remember to put your negatives in the right place or you're gonna…get the wrong answer again.'

Again, the emphasis appears to be on *what to do*, rather than on providing justifications for why the solution was correct. Later, she said that she asked questions in whole-class discussions because "I wanna know how each step goes or how to answer the question." She did not talk about giving help in terms of providing mathematical justifications, and rarely expressed a desire for the teacher or her peers to explain *why* their solution was correct.

Jordan also expressed an emphasis on procedures. An example can be seen in an interview regarding a video clip in which she was helping a group mate with a problem from her textbook (Hirsch et al., 2008, p. 336), reproduced in Figure 1. When I asked Jordan about the explanation she gave her classmate for this problem, it became clear that she was merely mapping the regions on the diagram to the terms in the equation $2x^2 + 7x + 6$: "I just told him that the two x squared was the two blue ones…plus the three green down there, and the four green up here, that's the seven x, that's the b. And then c was the, uh, the six yellow blocks." When I pressed her to explain how the picture showed the relationship between $(x + 2)(2x + 3)$ and $2x^2 + 7x + 6$, the only explanation she provided was the standard "FOIL" procedure (multiply the First, Outside, Inside, and Last terms, and add them together). This explanation, and that which she offered to her group mate, suggest that Jordan missed the point of the diagram, which was to use an area interpretation of multiplication to show how and why the FOIL procedure works ($2x + 3$ can be seen as $x + x + 1 + 1 + 1$, which is the length of the rectangle, $x + 2$ as $x + 1 + 1$, which is the height. Multiplying them gives regions that can be seen as having areas of x^2, x, and 1, and these regions can be separated into four parts to show the four products in the FOIL procedure).

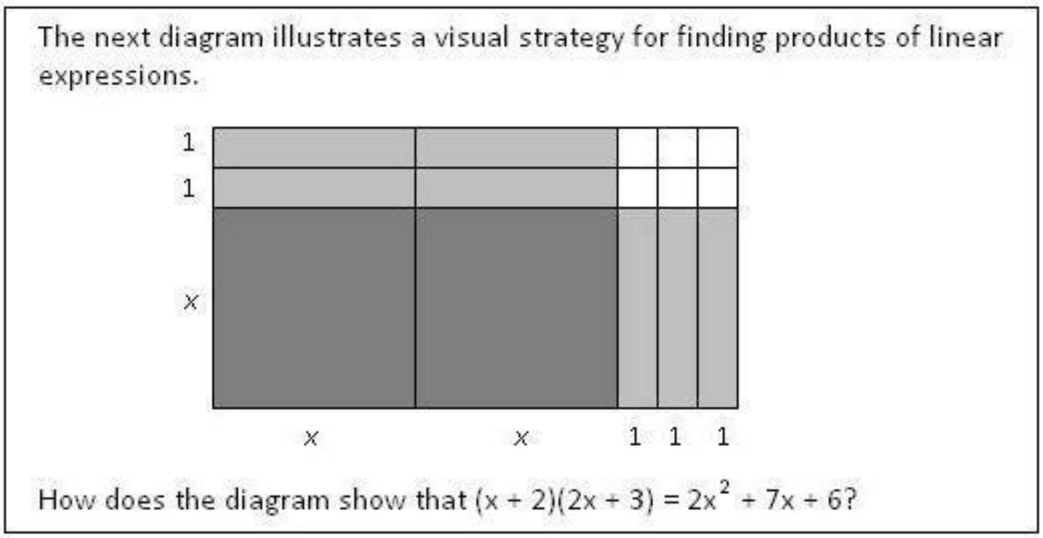

Figure 1: A task designed to reveal the conceptual meaning of binomial multiplication. Note: In the textbook, the large squares were colored blue, the rectangles green, and the small squares yellow.

Not only did Jordan fail to describe to her group mate how the areas were generated by the multiplication of the binomials, she also seemed to be satisfied with her explanation and did not ask the other members of her group about their solutions. This can be seen as an indication that knowing why $(x + 2)(2x + 3)$ is equivalent to $2x^2 + 7x + 6$ is either not important, or that the FOIL procedure is a sufficient justification for Jordan, although there was no evidence to suggest that her understanding of FOIL included anything more than the ability to complete the four steps it entails. This suggests that Jordan was satisfied with knowing the procedure for multiplying binomials, failing to press her group mates or herself for justification. This example is consistent with other comments in Jordan's interviews.

Discussion and Implications

The fact that both Sevanye and Jordan tended to describe working together in terms of passing knowledge between individuals could be related to the fact that they seemed particularly concerned with knowing procedures. That is, if mathematical understanding is primarily about being able to complete a procedure, then mathematical understanding is dichotomous—one can either do the procedure or one cannot. This dichotomy extends into the social fabric of the classroom. If you know the procedure, then you are likely to be positioned as a knowledge-giver. If you do not know the procedure, an efficient way to learn it is finding someone who knows it and getting them to explain it to you.

Thinking about mathematical understanding as grasping concepts and the relationships between them makes it possible to view collaboration in a way that does not position students asymmetrically. This perspective acknowledges that it is possible for one to understand quite a bit and still not be able to complete a procedure, and also for one to be able to complete a procedure with very little conceptual understanding. Mathematical understanding is much more complex than simply knowing what to do. Students are likely to have a variety of ideas, which, rather than being seen as correct or incorrect, could be seen as revealing different aspects of a mathematical concept. With this view, working together might be easier to see as a process of improving what is known collectively rather than a process of passing knowledge between individuals. Students might position themselves more symmetrically when working together. They might more readily accept that one not need be “knowledgeable” in order to contribute, and be more inclined to persist even in the absence of a knowledge-giving peer (or teacher).

On the other hand, if students see working together only as passing knowledge between individuals, this may affect the extent to which they are able to engage in truly collaborative work. In particular, students who take this perspective and consider themselves to be knowledge-givers have little incentive to work with their classmates. For Jordan, the possibility of reciprocation was an incentive for her to help her peers, but this incentive was not enough for Sevanye—perhaps because Sevanye more often was positioned in a knowledge giving role and saw fewer opportunities for reciprocation from her classmates. It should also be noted that, in light of the beliefs that both students seemed to share, Sevanye’s lack of a sense of responsibility for her classmates’ understanding is perfectly reasonable. If solid understanding (i.e., knowing a procedure) is a prerequisite for making contributions, and her peers rarely demonstrate this kind of understanding, then there is not much for her to gain by working with others.

One implication for teaching is that if Mr. Neal attempts to improve collaboration in his classroom by trying to help Sevanye be more like Jordan—to develop in her an obligation for helping others—then he may be missing the core problem. Perhaps the more fundamental issue is that neither student sees mathematics as a complex set of relationships that can be understood in many ways and from many perspectives, and thus they reject the possibility that others can contribute if they do not already know what to do. Simply putting students in groups and relying on goodwill (or the belief that one might at some point need help from others) may not be a reliable strategy for encouraging consistent collaboration in the classroom.

Teachers might address this problem by paying attention to the way they portray mathematical knowledge and what it means to work together. Webb, Nemer, and Ing (2006) suggest that students are likely to adopt the helping behavior that is modeled by their teacher. In particular, if the teacher models knowledge-giving behaviors and emphasizes procedures, then students may not be challenged to see working with each other in less asymmetric ways. In this class, Mr. Neal did not often directly help students during group work; however, during whole-class discussions, he did resolve issues of correctness and provided explanations for problems that students had worked on. A more detailed analysis of Mr. Neal’s teaching practices might reveal more about how they could have been adapted to challenge students’ beliefs about collaboration and mathematical understanding.

Conclusion

In this paper, I used the perspectives of two high school students in the same mathematics class to argue that the beliefs students have about the nature of mathematical understanding and the purpose of group work can affect the extent to which their participation in "collaboration" is truly collaborative. Promoting collaboration means more than making sure that students adopt responsibility for helping other understand; it means addressing and challenging these underlying beliefs.

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