Home Page JSR

# Dynamics and Geometry of the Human Sexual Response

**H.M. Hubey**

**Department of Computer Science, Mathematics, and Physics**

**Montclair State University**

**Upper Montclair, NJ 07043**

**hubey@pegasus.montclair.edu**

## Dynamics and Geometry of the Human Sexual Response

**H.M. Hubey**
**Department of Computer Science, Mathematics, and Physics**
**Montclair State University**
**Upper Montclair, NJ 07043**
**hubey@pegasus.montclair.edu**

### Abstract

A nonlinear differential equation model and its associated catastrophe is shown to model the simplest version of the sexual response of humans.

### Keywords

sexual response; differential equations; catastrophe theory; nonlinear damped harmonic oscillator; differential geometry; oscillation; wave equation; Schrodinger Equation

### Systems Modeling

"The simplest definition of a system is that it is an integrated assembly of interacting elements, designed to carry out cooperatively a predetermined function. Systems Science, then is the scientific explanation and theory of systems in the various sciences (e.g. physics, biology, psychology, economics) " [Hubey1979]. It is unfortunate that the study of biology did not approach that of the inanimate physical sciences for a long time. It still hasn't, although it has made remarkable progress in recent decades in all areas, including mathematical modeling of genetics [Hubey,1996,Kojima,1970, Roughgarden,1979], theoretical biology [Brooks & Wiley, 1986, Schmidt-Nielsen,1984]], consciousness and intelligence ignoring some books on artificial intelligence[Dennett,1987, Ornstein,1975], even in the area of brain research [Durden-Smith1,983, Eccles,1989, Jerison,1973, MacLean,1973, Restak,1979, Sagan,1977]. One of the great sucess stories of science and mathematics was the development of calculus. Unfortunately, its weaknesses were not apparent until recently. Not every process in the world is continuous and slowly changing. The particular combination of continuous (and often slow) changes vs discontinuous (rapid) changes (effects) can probably be best described/modeled in terms of a new theory invented byThom[1975] and which has benefited from the many contributions by E.C. Zeeman [1977]. This branch of mathematics is known, appropriately enough, as Catastrophe Theory. The literature is replete with scientific papers applying catastrophe theory to various social, psychological and biological phenomena [for example, Zeeman,1977, or Woodcock & Davis,1978, Jackson,1991]. Many more books have been written on nonlinear systems and complexity in recent years[see for example Hubey,1996, Guckenheimer& Holmes,1983, Bell,1990, Drazin,1992, Zurek,1995]. Catastrophe theory is in some ways the ideal tool to analyze phenomena with rapid or discontinuous changes, and has been used for various biological models by Thom [1975]. The purpose of this paper is to develop a mathematical model, however primitive, of human sexual response.

**The Facts and Models**

Perhaps the best way to start would be to present a phenomenological model since the justification of the model is difficult to construct using the traditional analytic-synthetic methods. We start with the known stylized facts or commonly held knowledge or observations; to them we attach observations that correlate. For example, when Fahrenheit noticed that a liquid like alcohol in a glass tube went up with increasing temperature, he merely connected the observation with the common observation 'felt' by everyone that the temperature went 'up'. All science starts at the perceptual level. When it does advance it starts to measure things with instruments instead of the naked senses. The correlations of the phenomena with measurements have to match what can be observed and felt with out naked senses. To go deeper would bury us in the realm of the bottomless pit of philosophy of consciousness and perception [Dennett,1995 , Ornstein,1975, Searle,1984, Skinner,1974].

The word *tension* (or sexual tension) is used in everday speech and by which we do not mean either muscle tension nor a strain gage measurement of the penis. Every one knows what it means but finds it difficult to define much less measure. Both in axiomatic mathematical systems and in physics there are undefinable quantities which we assume we know. In this sense the concept *sexual tension* will be used in the same way, although we might suggest some means of measuring it later. Every science goes through this stage of being forced to work with undefined quantities. We read, this time from Masters and Johnson; "The basic physiological responses of the human body to sexual stimulation are two fold in character. The primary reaction to sexual stimuli is widespread vasocongestion, and the secondary response is a generalized increase in muscle tension. The vasocongestion may be either superficial or deep in distribution, and the myotonia reflected by voluntary or involuntary muscle contractions." [Masters & Johnson, 1966:7].

Further hints that point in the proper direction can be obtained from the same book: "Only one sexual response pattern has been diagrammed for the male. Admittedly there are many identifiable variations in the male sexual reaction. However, since these variants are usually related to duration rather than intensity of response, multiple diagrams would be more repetetive than informative. Comparably, three different sexual patterns have been diagrammed for the human female. It should be emphasized that these patterns are simplifications of those most frequently observed and are only representative of the infinite variety in female sexual response. Here, intensity as well as duration of response are factors that must be considered when evaluating sexual reaction in the human female." [Masters & Johson, 1966:5]. The state of scientific study of sex since then can been summarized as:

> The occurrence of relatively intense physiological arousal as a frequent concomitant of sexual response naturally focuses attention on the autonomic and somatic manifestations of this emotional state. Thus, in the tradition of Masters and Johnson, it is not unusual for accounts of sexual arousal to enumerate, extensive changes in cardiovascular, respiratory, vasocongestive, muscular, and other physiological activities occurring in response to~sexual stimulation. While this approach has been useful in describing the widespread physiological reactions occurring during sexual arousal, the result has been a relative neglect of cognitive-affective processes and the potential interaction of these response dimensions....The emphasis on peripheral physiology has also included a focus on the importance of orgasm as a pivotal component and the culmination of the sexual arousal process. Rosen [1988:25]

Specifically, the modifications/additions/criticisms that have been proposed since then can be all compared with the 4-stage Masters & Johnson standard and can be grouped into several classes:

*(i) criticism of the taxonomy of the four-stages:* included in this are (a) Robinson [1976] who has drawn attention to the apparently contrived separation of sexual response into four discrete stages, especially the distinction between the excitement and the plateau phases; (b) Kaplan [1977,1979] who noting the relative neglect of motivational and psychological factors in the Masters & Johnson model, has produced a three-stage model of desire, excitement, orgasm. A key postulate of this model is these three phases are mediated by *separate* and *interrelated* neurophysiological mechanisms; the first (desire) is generated by central mechanisms( limbic activation), whereas excitement and orgasm are connected with the stimulation of peripheral reflex pathways in the lower spinal cord. However a more exact specification of anatomical sites and physiological mechanisms are still missing. The hypothesis that desire is a necessary precondition for the occurrence of excitement and orgasm is contradicted by evidence from multiple sources [Rosen & Beck,1988]. It should be noted that Kaplan could have been included in the next category as well.

(ii) *criticisms of the relative neglect of the psychological degree of freedom or the relative lack of synthesis of cognitive-affective states with the physiological processes of sexual arousal:*

Byrne [1977,1986] attempts to provide a more complete description (like Kaplan) and gives a description of the sex act as multidetermined sexual behavior sequence in which three cognitive-affective dimensions are conjectured to mediate the effects of erotic stimulation on physiological arousal, such as imagery processes, such as memories, informational responses encompassing beliefs, knowledge, and expectations concerning sexuality, and affective responses such as judgments, feelings, and other subjective perceptions. Byrne [1983] has also posits a key mediating parameter of affective reaction, termed "erotophobia-erotophilia", which refers to a generalized evaluative-emotional response to sexuality in which erotophilia is said to represent a positive response to sex and negative erotophobia, a negative one. So far this amounts to not much more than making up names for bundles of psychological observations however the fact that Byrne even thinks of using both positive and negative numbers augurs well for possible mathematical modeling. This affective response has been shown to influence individuals' willingness to experience erotic stimulation [Byrne, Jazwinski, DeNinno, & Fisher, 1977]; the duration and intensity of erotic fantasies [Moreault & Follingstad, 1978]; and the degree of subjective arousal and perception of genital sensation in response to erotic stimulation [Mosher & O'Grady, 1979]. According to Rosen and Beck [1988], "this model represents the most comprehensive attempt to date to explain relevant interactive mechanisms of sexual arousal." Yet another interactive model has been proposed by Barlow [1986] which focusing largely on the development of sexual dysfunction, describes posited sequence of interactive processes consisting of: (a) external events such as erotic stimuli, and specific partner behaviors, (b) internal mediational variables, which subsume transitory emotional responses, a (c) more enduring cognitive-affective reactions, and the full range of behavioral outcomes.. The results of these holistic attempts can be summarized as:

> In the tradition of the cognitive arousal theory of emotion, we assert that the interactions among several dimewns of arousal, inch the physiological, cognitive, affective, and motttional component crucial for defining a response as sexual. In contrast to the original formulations of Schachter and Singer [1962], however, we would emphasize that sexual arousal cannot be viewed as arising out of a state of diffuse or nonspecific (sympathetic) activation, given the stimulus-response specificity of genital vasocongestion. With reference the salience of cognitive attributional processes in defining emotional experiences generally [e.g. Reisenzein, 1983], however, we find that much of the theory and research on sexual response has neglected this all-important dimension. From our perspective, sexual arousal cannot be defineddequately without highlighting the critical role of cognitive labeling and subjective experience in determining the response to a given stimulus as sexual. Ultimate would hold that an individual's awareness or self-report of internally experienced arousal is primary in defining a response as sexual, irrespective of the nature or extent of his or her physiological reactions. by Rosen and Beck [1988:28]:

(iii) *identification" of the "dimensions of sexual response:* in which Bancroft [1983] describes four "degrees of freedom" of sexual arousal: (1) sexual appetite or drive; (2) central arousal; (3) genital responses; and (4) peripheral arousal. There seems to be some evidence that these are almost independent or orthogonal. Included in the category of sexual appetite or drive; according to Bancroft, are both motivational factors (e-g, "libido") and sexual arousability are included in the category of sexual appetite or drive (and it's not clear if these are genetic or again environmental). Whalen [1966] is of the opinion that sexual arousability is a degree of freedom (DOF) separate from desire, and and is seen as the individuals propensity for arousal given an adequate source of sexual stimulation. [Rosen & Beck,1988:33].

## The Mathematical Model and Its Precursors

Among the models that are of interests for the mathematical model in this paper are (a) the dual-innervation model of Weiss [1972] for the neurophysiological basis of erection in which " separate pathways are proposed for psychogenic and reflexogenic control of erection. ... according to this model, the reflexogenic component consits of a sacral parasympathetic pathway mediated via the nervi erigentes, while the psychogenic pathway is sympathetic in origin, with efferent innervation descending from the thoracic-lumbar portion of the cord." [Rosen,1988:58]; and (b) the two-phase model of male ejaculation by Davidson [1980], another variant of the two-factor model psychophysiology of orgasm, termed the "bipolar hypothesis." According to this theory, the sympathetic discharge associated with the emission phase invariably results in "sexual satiety," and is therefore the basis of the refractory period in males. In contrast, the synchronous contractions of the pelvic skeletal musculature associated with the expulsive phase are said to be accompanied by the central effects of "orgasmic experience without satiety." So far nothing can be said as yet about the location or nature of the putative neural centers which triggers the bipolar reactions. This is a two component model like the others, and has been used to explain several important phenomena. It "appears to be on relatively firm ground in accounting for suppression of exual arousal following sympathetically innervated seminal emission. the fact that women do not experience a corresponding physiological response during orgasm may explain why the refractory period seems to be invariable for men but not for women" [Rosen & Beck, 1988]

It should be mentioned that traces of "bipolarity" or the "opponent processes" of psychology (which seem to represent trade-offs between either two clearly discernible variables or measurements which are the opposite sides of the same coin, so to speak) can be seen in Bancroft's ideas in which the central arousal, Bancroft's second component of sexual response, refers to central nervous system (CNS) activation and attentional factors that underlie psychological processing of sexual stimuli In contrast, the third and fourth components described by Bancroft, genital responses and peripheral arousal, have received the most extensive discussion to date [e.g., Masters & Johnson, 1966] and are generally the least controversial [Rosen & Beck, 1988: ]. The fact there are at least two components (in a grossly aggregated way) has always been known by almost everyone, and that it was also clearly delineated by Masters & Johnson: "At the outset it should be made perfectly clear that although stimuli are characterized as somatogenic or psychogenic in origin... this does not imply that any form of stimulation is or can be purely somatogenic in character. All stimuli are appreciated, delineated, and referred by higher cortical centers." [Masters & Johnson,1966:61].

## Summarization of the Mathematical Model

Figure I is a reconstruction [from Masters & Johnson] of the male sexual response. The corresponding one for the female is more complicated and will probably have to be constructed as a combination of the solutions for the simpler case. It is clear that the abcissa in Figure I is time. Lest it be incorrectly believed that the model being developed is unrealistically and woefully inadequate we can again refer to: " However, again and again attention will be drawn to direct parallels in human sexual response that exist to a degree never previously appreciated. Attempts to answer the challenge... have emphasized the similarities, not the differences, in the anatomy and physiology of human response." [Masters & Johnson, 1966:8]. A cusp catastrophe, Figure II, has essentially the same information as in Figure I, but time is implicit; the abscissa is a control parameter. The second hint comes from the nonlinear damped harmonic oscillator differential equation given below:

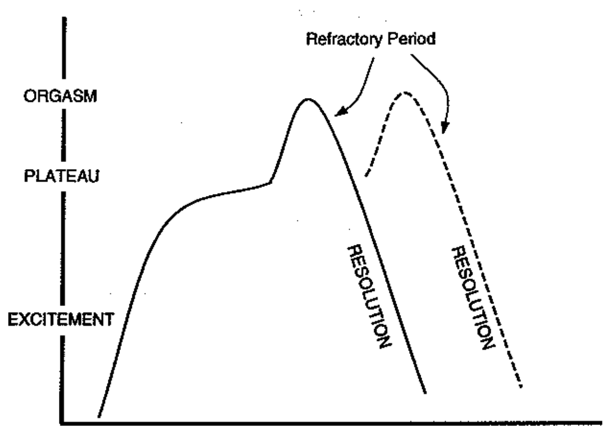


Figure I

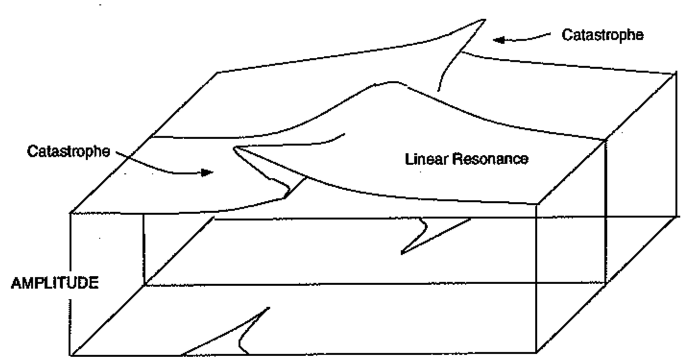


Figure II

1) $$d^2\Psi/dt^2 + 2\xi d\Psi/dt + \omega_n^2\Psi + \mu\Psi^3 =$$

$$=\{d^2/dt^2 + 2\xi d/dt + \omega_n^2\}\Psi + \mu\Psi^3 =$$

$$= L_0\Psi + L_1\Psi = A\sin(\Omega t)$$

where $\xi << 1$ and $L_0$ and $L_1$ are the linear and nonlinear operators respectively. The source term in the differential equation above is the simplest kind of rhythmic (periodic) physical stimulation. The nonlinear differential equation above cannot be solved in closed form. Solutions of this equation are usually approximate. There are various methods that have been used to obtain qualitative knowledge about the solution behavior of the system [see for example Minorsky,1962, Shaw,1988]. One such solution method consists of finding the relationship of the solution to the forcing function. In the linear case, it is known that resonance occurs at that frequency which equals the natural frequency of the system, and that the amplitude of the system tends to infinity at resonance. It is already known from the physical (mechanical and electrical) systems that the nonlinear system exhibits what is named the jump phenomenon. In Figure II, the response surface is given. It should be noticed that this is for the case of $\mu<0$, $\mu>0$ and $\mu=0$. The last case corresponds to the linear model and refers to case. Figure III is a cross section from Figure II and it clearly shows the jump phenomenon. If a nonlinear system is driven by a sinusoidal forcing function the response of the system (that is the amplitude) is

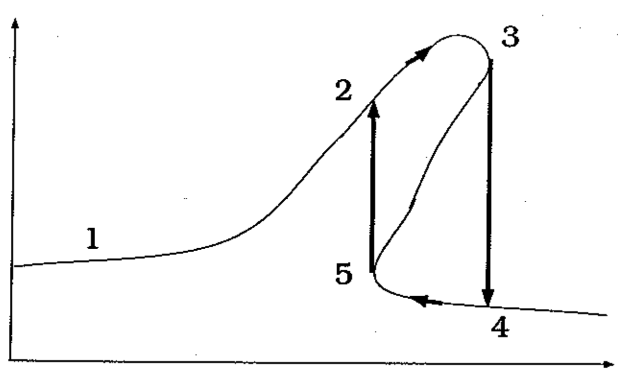


Figure III

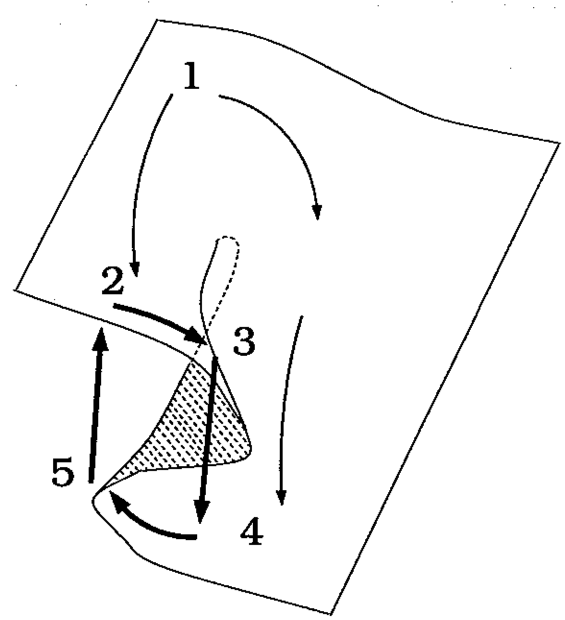


Figure IV

as shown in the figure. As the frequency is increased, at first the amplitude of the response increases [say from 1 to 2]. Then as the frequency is increased it reaches a peak [at 3] at which it suddenly drops [to 4]. If on the other hand, the frequency is decreased, at first the amplitude increases slowly[say from higher frequencies than to 4] but there is no jump at 4 upt to 3. Instead it follows the lower curve up to 5 at which there is a sudden jump to 2 and the system starts to oscillate at the higher amplitude. Thus the magnitude of the amplitude, which we assumed (or modeled) to represent sexual tension first increases slowly and then suddenly goes through a discontinuous jump through several cycles as shown in Figure III. This has been confirmed experimentally in mechanical and electrical systems. This cycle [3-4-5-2-3-4...] as shown in Figure III is the orgasm phase. However this does not always have to happen, as is well known. According to Masters and Johnson: "In this phase sexual tensions are intensified and subsequently reach the extreme level from which the individual may move to orgasm. The duration of the plateau phase is largely dependent upon the effectiveness of the stimuli employed, combined with the factor of individual drive for culmination of sex tension increment. If either the stimuli or drive is inadequate or if all stimuli are withdrawn, the individual will not achieve orgasmic release and will drop slowly from plateau-phase tension levels into an excessively prolonged resolution phase." [Masters & Johnson,1966:6] "Physiologic residuals of sexual tension usually are dissipated slowly in both the male and female unless and overwhelming orgasmic release has been experienced." [Masters & Johnson,1966:7] This last sentence is indicative of yet another possible resolution of sexual tension which can be seen in Figure IV. In other words, another path can be taken on this surface due to psychogenic changes, which, of course implies that the idealized version of the equation above with constant coefficients which represents the normal or common response surface is not always valid and that a more realistic model will have time-dependent coefficients so that the response surface can change in time due to psychological effects. Furthermore, we know that for the linear case (i.e. $\mu=0$) there is no jump but there is a resonance phenomena which is mathematically a singularity, and since the response surface does not have the tilt but rather the common DHO response, there is no orgasm but rather a slow decrease of the amplitude with increasing driving frequency. Hence the nonlinearity coefficient, $\mu$, can represent a particular type of disfunction in which there is failure to achieve orgasm despite sustained excitation.

## Phenomenology and Epistomology: Layers of Description

One of the problems with mathematical modeling of psycho-social phenomena is that the derivations are not entirely clear since it deals with poorly understood phenomena in which we are not even sure of what we measure if such measurements even exist. It might also be argued that the conceptual basis for catastrophe theory and rationale for application to human sexual behavior are unclear. Other criticisms of the model may be that the relevance and practical implications of the approach are lacking. Relevance and practical applications are not germane to the discussion of any scientific work since either science is like art and it is done for the same reason as people climb mountains (i.e. for its own sake) and one should look at whether the equations model reality to some acceptable degree of accuracy or consider that all science in the end is useful. It would take us too far from the specifics of the model to epistemology and ontology to answer these questions. Application to human sexual behavior can be clarified and its shortcomings discussed after the full model is developed. The only germane problem is to more clearly explain the reasons as to what the model is and what it represents. In very developed scientific fields such as physics, one always tries to explicate phenomena in terms of already well-tested and well-known fundamental principles. In a field such as mathematical biology, the charge of phenomenology cannot be leveled at mathematical models since they almost all consist of such models. The only problems are with the understanding of the model. Probably the best thing that can be done is to give analogies to what the equations represent and similar problems faced by researchers faced by other fields. It would seem, ironically, that Skinner did, after all have the right idea [Skinnner,1974] in that there is negative feedback in the system and that the system responds to the forcing in this manner. Indeed, the *Black Box* idea which Skinner borrowed from physics has been in use in the social sciences, even by Skinner's opponents, for example Chomsky. The performance-competence distinction of Chomsy makes implicit use of the Black Box [see Hubey,1994]. Similar problem of potential-actual plagues the intelligence measurement game [see for example Hubey,1996, Hubey,1995]. There are various facets to geometry and the levels of descriptions of events in the real world. Zeeman [1988] has identified six levels especially with regard to biological and psychological phenomena: (1) singularities, (2) fast dynamic (homeostatis) , (3) slow dynamic(development), (4) feedback, (5) noise, and (6) diffusion. The first layer has to do with the catastrophe theory of Thom; layers

2-4 involve ordinary differential equations; layer 5 with stochastic differential equations, and layer 6 is concerned with partial differential equations. As can be seen in the examples below, phenomena may be examined at various layers and we may also be able to see only various views of it as in the story of the blind men and the elephant. Complex phenomena can also be seen at various levels; for example, for geometry we have, the topological, the differential, the algebraic and the affine views.In this sense, then Thom's catastrophe theory is an addition to our toolkit and the model presented in this paper is a holistic one in that it is a complete description taking into account both psychological and physiological parameters although it also offers only glimpses of various views of an extremely complex phenomenon which is yet to be fully understood. In that sense it is a very simple model indeed, although an almost complete one; that is, it is a top level description as are all others.

**Oscillation Models in Physics and Biology**

It is a remarkable fact that almost all of physics is explicable in terms of a few equations. One of them, perhaps the most important one is the family of equations that fall under the heading of oscillation; these equations are the wave equation, Helmholtz equation, the damped harmonic oscillator equation, the Duffing equation, and equations such as the string equation, the beam equation etc. Probably the most general form of the equation for a whole class of equations is the wave equation which is of the form

2) $$\{\nabla^2 + \partial^2/\partial t^2\}\ \Psi(\mathbf{r},t)= f(\mathbf{r},t)$$

where $\nabla^2$ (which is equal to $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ in Cartesian coordinates) is the Laplacian operator, the $\partial^2/\partial t^2$ is the second derivative with respect to time, $\Psi(\mathbf{r},t)$ is the wave function (where r is a space vector and t is time) , and $f(\mathbf{r},t)$ is the forcing or the source term. This equation describes among others the propagation of electromagnetic, and acoustic waves. If we take a temporal Fourier Transform we then obtain

3) $$\{\nabla^2 + \omega^2\}\ \Psi(\mathbf{r},\omega)= f(\mathbf{r},\omega)$$

is called the Helmholtz (reduced wave) equation. The single dimensional version of equation(2) is

4) $$\{\partial^2/\partial t^2 + \partial^2/\partial x^2\}\ \Psi(x,t)= f(x,t)$$

This equation is used to provide simple models of oscillations for, among other things, the air column in a tube, or vibration of a string. The damped version of the same is given by

5) $$\{\partial^2/\partial t^2 + 2\xi\cdot\partial/\partial t + \partial^2/\partial x^2\}\ \Psi(x,t) = f(x,t)$$

If we take a spatial Fourier Transform we then obtain

6) $$\{d^2/dt^2 + 2\xi\cdot d/dt + \omega_n^2\}\ \Psi(k,t) = f(k,t)$$

which is the damped harmonic oscillator. This equation is probably the simplest oscillator model (except for the undamped version) and describes in general things such as the oscillation of a mass attached to a spring, the oscillations of the currents in tuner circuit in a radio or TV receiver, oscillations of the insulin-sugar mechanism, and the oscillations of a bound electron. It is also used to model the oscillations in the oral cavity which produces speech and has been used in digital form to produce filters that produce digital synthesized speech such as in MITtalk. The model given in eqns (1), in addition, has a cubic nonlineary added and is known in the literature as the Duffing oscillator. This equation has been used to model oscillations of strings in string instruements and also used to explain the quantal aspects of speech [Stevens1989, Hubey1994]. It is also one of the fundamental catastrophes [Thom,1975]. The basic model analytically can be thought of as something like the opponent processes of psychology which seems to reflect most of all the fact that there is negative feedback loop since all controllable systems must exhibit such a capability. Suppose that sexual excitement and orgasm is mediated by a two-factors (which are still unkown) as already conjectured by Davidson, Bancroft, Kaplan etc. We can start our modeling by assuming the simplest kind of a dynamic dependence, by which we mean a coupled set of ordinary differential equations using two variables, as below

7) $$U' = F(U,V) + N(t)$$

8) $$V' = G(U,V)$$

This is a very general model except that the changes in U and V are dependent on both U and V and that the function N(t) is some external excitatory "force" which is a function of time. We assume that there is some kind of static or dynamic stasis (or equilibrium) normally in that we can write the variables as $u=U-U_0$ and $v=V-V_0$ where the $U_0$ and $V_0$ are the *equilibrium* values. This implies that $F(U_0,V_0)=0$ and $G(U_0,V_0)=0$. Then we obtain

9) $$u' = F(U_0+u,\ V_0+v) + N(t)$$

10) $$v' = G(U_0+u, V_0+v)$$

Now, we can see from the Taylor expansion that

11) $$F(U_0+u, V_0+v) = F(U_0,V_0) + u\cdot\partial/\partial U\{F(U_0,V_0))\} + v\cdot\partial/\partial V\{F(U_0,V_0))\} + \varepsilon_1$$

12) $$G(U_0+u, V_0+v) = G(U_0,V_0) + u\cdot\partial/\partial U\{F(U_0,V_0))\} + v\cdot\partial/\partial V\{F(U_0,V_0))\} + \varepsilon_2$$

where $\varepsilon_i$ are small compared to u and v. Therefore assuming that the deviation of U and V from $U_0$, and $V_0$ is small (and thereby making an approximation) we see that

13) $$u' = -m_1 u - m_2 v + N(t)$$

14) $$v' = -m_3 v - m_4 u$$

where $m_1 = \partial/\partial U\{F(U_0,V_0))\}$; $m_2 = \partial/\partial V\{F(U_0,V_0))\}$; $m_3 = \partial/\partial U\{F(U_0,V_0))\}$; $m_4 = \partial/\partial V\{F(U_0,V_0))\}$

where we've assumed that the signs of the partials are negative because of the negative feedback effect that is necessary for stability of systems. We can simplify the equations by differentiating (13) to obtain

15a) $$u'' = -m_1 u' - m_2 v' + N/dt$$

and subsituting for v′ from (14) and solving for v in (13) and also substituting for v in which we obtain

15b) $$u'' + (m_1+m_3) u' + (m_1m_3+m_2m_4)u = m_3N(t) + dN/dt$$

which is the damped harmonic oscillator since it is of the type as in eq. (6). We note that this equation is still linear i.e. it's the linear version of eq. (1). We note that the forcing is still sinusoidal (which is the simplest approximation of a periodic stimulation or excitation.) We could have just as easily defined N(t) as an integral of a sinusoidal function since the state of sexual excitation is a function of the past excitations or we could have made N(t) a convolution integral with some kind of weighting. Only the future experiments will tell which of these is the best model since about the only thing we know now is that it is essentiallly periodic. The spectral response of the linear harmonic oscillator , as is well known, has a high response at the natural frequency $\omega_n$, and that the nonlinear response for a small cubic nonlinearity (i.e. eq.(1)) is derived on the basis of an approximation from the linear case.

### *Slow-Fast Dual Mechanism*

There exists yet another derivation for the model. That the lower life forms (at least the males) can have something like orgasm would indicate that the primitive part of the *triune brain*[ McLean,1973] is involved and that this would indicate the limbic system. Furthermore that there can be emission without orgasm (refs) is indicative that there seems to exist two separate systems that work together to produce orgasm and that another approach to the derivation of the mathematical model would be fruitful. We can take an approach similar to that pioneered by Zeeman [1977] in that we can posit the existence of a fast equation and a slow equation which work together to produce catastrophes. Following the ideas of Zeeman [1977] the model can be written as a system of two first degree equations

16a) $$u' = -\alpha v$$

16b) $$v' = av + bu + cu^3 + d\sin(\Omega t)$$

The set above is equivalent to the single second degree differential equation given in eq.(1). We can differentiate eq. (16a) and substitute for v′ (from 16b) to obtain eq.(1) with $\omega^2 = \alpha b$, $2\xi = \alpha a$, $\beta = \alpha c$, and $A = \alpha d$. The constant $\alpha$ is large so that eq.(16a) is the *fast equation* [see Zeeman,1977]. The constants a,b, and c are small so that eq.(16b) is the *slow equation*. Hence we have $\xi < 1$, $\omega$ can be large, and $\beta$ is also small. The linear version (i.e. c=0) is equivalent to the set of equations (13-14) with $\xi = (m_1+m_3)/2$; $\omega_n^2 = m_1m_3+m_2m_4$; $\beta = 0$ and with the substitution of an oscillatory forcing or excitation $M\sin(\omega t) = m_3N(t) + dN/dt$. It can be shown that the set of equations below

17a) $$u' = -2\xi u - \alpha v$$

17b) $$v' = bu + cu^3 + d\sin(\Omega t)$$

is also equivalent to eq.(1) with b, c and d given as above. The same reasoning as above shows that the first equation is again the fast equation and the second, the slow equation. What is important in these models is that as two one dimensional equations they are both of the standard/classical *opponent theory* of psychobiological processes, of which the bipolar theory, and the somatogenic and psychogenic response type, as in Masters and Johnson or Kaplan or Bancroft. It can be seen from the equations that the psychogenic (and some somatogenic) effects are being represented by the coefficients of the equation while sexual tension can be approximated as the magnitude of the amplitude of the response. This also has some evidence to back it up in that when the trigger occurs before some full orgasmic platform/state has been reached it is not satisfactory and that this seems to be connected with the muscle stretching and firing (refs). That means that a correlated opening up of the valve allowing blood to spurt into the say, penis, would tend to increase the muscle tension so that in the fully developed state (i.e. at point 3], orgasm would follow whereas, high frequency excitation, say using a vibrator, might trigger orgasm-like reactions (mini or micro orgasms in which there could be a trigger with the concomitant blood spurt) but it would fail to trigger orgasm since the response surface has not yet fully developed. This would indicate that the response surface slowly develops via psychogenic and somatogenic excitation, and that purely physiological excitation could trigger only a single component of a multiple complex system. Furthermore it would add more evidence to the view that this response surface is not really a part of the system (as in mechanical or electrical systems) but that it only develops during the excitation phase so that the coefficients themselves are functions of the organism, and its stimulation. Hence we can see that, using time as a proxy (to simplify) for these as yet unmeasurable variables, we should make the coefficients functions of time which immeasurably complicates the equation. The equation, as complex as it is these days (since we have no means of solving it directly) is only the simplest kind of a mathematical model of an immensely complex behavior. Of course, the fact that the catastrophes are a high level (i.e. simplified views) descriptions has already been noted [Zeeman,1977]. As for experimental evidence, and for thoughts on the subject by other researchers, we note that Kaplan [1979], for example, considers the emission phase to be controlled entirely by sympathetic innervation in particular, alpha-adrenergic receptor stimulation of the smooth muscles in the male accessory glands. According to Kaplan, the emission response is experienced subjectively as "ejaculatory inevitability," but is not intrinsically pleasurable. Ejaculation, on the other hand, is controlled by somatic innervation of striated muscle groups at the base of the penis, find is accompanied by "the typical pleasurable orgastic sensations" [1979: 20]. Kaplan also postulates the existence of an "orgasm center" in the sacral spinal cord, which is said to coordinate both phases of the response. As for the mechanism of trigger, researchers have specified myotonic or vascular responses as orgasm triggers. Sherfey's proposal [1966] is that orgasm is a spinal reflex that is triggered by firing of the stretch receptors in the pelvic musculature, while Mould[1980] suggests that a principal effect of vasocongestion is to cause biasing of the gamma fusimotor muscle spindles. Once the muscle spindles become highly biased, and a dynamic stretch reflex is initiated in the alpha fusimotor. The chemical basis specifically has been investigated by Brindley [1983] who has shown that alpha-adrenoceptor blockers produce reflex erections. It has also been shown that the injection of papaverine also causes erections by [Virag & Virag 1983, Zorgniotti & Lefleur 1985]. Naturally the biochemical aspect if a part of the whole phenomenon of sexual excitation and should be accounted for (eventually) by mathematial equations, but the ordinary differential equation model shown here and its associated geometry (catastrophy) is a high level description and cannot account for low level phenomena. When more is known, the relationships of the coefficients of equation (1) to the lower level biochemical, mechanical and electrical underpinnings may be established.

As for the reported marked alterations in consciousness in the literature, explanation proposed by Davidson [1980], is again a two-factor model, which not surprisingly has to do with rapid fluctuations or oscillations in autonomic balance from parasympathetic (trophotropic) to sympathetic (ergotropic) dominance andthat this is the cause of the major alteration in conscious experience. Drawing upon Gellhorn's [1957] original hypotheses on the effects of autonomic "imbalance" on central arousal states Davidson suggests that high levels of sexual arousal result in simultaneous (i.e., "nonreciprocal") activation of both autonomic systems, and a consequent change in central arousal. The word "imbalance" is obviously an attempt at description of changes in the equilbrium of a system or fluctuations or oscillations between two poles or two phases. Gellhorn's model would also predict a rapid "rebound" effect following strong ergotropic activation which may account for the state of quiescence usually associated with the refractory period. Finally, Davidson notes that major changes in autonomic balance appear to be associated with a shift to right (nondominant) cerebralactivation, which is consistent with the results obtained by Cohen et al 1976]. In the mathematical model this would correspond to the unfolding of the response surface so that it is flat, and thus the disappearance of the orgasmic platform (or curved sheet as shown in the figures). Just as the coefficient $\mu$ controls the ability to hold tension and to raise it to higher levels before release, the other parameter $\omega n$ is the natural frequency which falls presumably for all humans in the same range. Then the coefficient $\xi$ is a control parameter which probably has something to do with the erotophobia-erotophilia variable of Byrne, since the response is greater for small values of $\xi$. Appropriately enough, this parameter in physical systems is the amount of "damping" in the system so that high values will dampen down the oscillations and hence the response even at resonance. For values of $\mu=0$, the system is linear and an increase in the frequency of the excitation cannot cause a jump phenomena. Instead we will see a continuous increase in amplitute (which we've claimed represents sexual tension) until resonance and the response will decrease if the frequency is increased, so that there could be a slow decrease in tension but no orgasm.

## Conjectures, Improvements Needed: A More Realistic Version

It is well known that one of the primary forces that produced the twentieth century revolution in physics was quantum mechanics, and the Schrodinger Equation occupies a place of central importance in this field. Strangely enough not only is it a form of the wave equation but it also suffers from the same difficulties as the model in this paper. For example, the time-independent Schrodinger equation given by [see for example Cushing1975, or Butkov1968] with the potential term V(x)=0;

18) $$-(h/2m)\nabla^2 \Psi = ih \cdot \partial\Psi/\partial t$$

is, as can be seen by comparison to the earlier equations, a wave-like equation except that it is complex valued, and thus only has an interpretation for $|\Psi^2|$ (i.e. the modulus or magnitude) as a measure of probability that a particle will be near that point. Thus in this sense the wave function $\Psi$ provides a statistical connection between the wave and the associated particle; it tells where the particle is likely to be, not where it is. In the models for orgams in this paper the magnitude of the amplitude $|\Psi|$, not its sign that is significant.It should be noted that it is through the use of the imaginary values i=sqrt(-1) that oscillatory solutions can be obtained for the first degree differential equation given in eq.(18). We can write the harmonic oscillator (with $\mu=0$) as a first degree equation as $x'=i\omega x$. We have a similar problem with the modeling of the sexual response with nonlinear differential equations and catastrophe theory. In one way, it is merely the mathematical formulation of many of the ideas already expressed in words; in another way, it is predictive since we can try to extrapolate from the equations as to what kinds of things to expect if the model is correct. Since every model is restricted to a particular set of constraints and domains, only time and further experiments and advances in measurements and other branches of the biological sciences can tell if the model will withstand the test of time. As far as the simple model of this manuscript, and its implications, it is abundantly clear that much more work needs to be done, including but not restricted to experimenting, conducting correlation-regression analyses to estimate the coefficients (the parameters $\xi$,$\mu$, and $\beta$) and further developing the underlying mathematical model. What the present model cannot show is that it represents the case for constant coefficients whereas in real life it is possible that these parameters change slowly throughout the duration of sexual intercourse so that the response surface could be changing due to psychogenic factors. The equations are already difficult enough to solve with constant coefficients so that considering the coefficients to be functions of time would make the equation intractable. Indeed reality is so much more complex since we already know that the coefficients must be functions of the *brain state* or the *mind state*. All of this assumes that there are no physical or biological problems or defects, and if there are we assume that there will be appropriate adjustments in the coeffients. We can see that if the coefficient $\mu$=0 or is very small then the response surface is equal to that of the linear oscillator and that if this happens during intercourse, instead of the usual orgasm there can occur a slow dissipation of tension instead of the common orgasmic release. Although the time is implicit in the response surface, the common notion of thinking about other things to hold off the moment of release or the converse of concentrating to hasten it, which has now been experimentally confirmed [see Geer and Fuhr[1976] who examine the influence of cognitive distraction on peripheral arousal [penile tumescence) in college-aged males, and Farkas [1979] who studied the interactive effects of distraction and performance demand on tumescence] have to do with changes in the motion of the phase point on this response surface so that the surface cannot really stay constant. In this case there's an analogy to yogic-like control of heart via breathing which seems to be both automatic and volitional. As has been known for centuries among mystics, control of breathing [which seems to be a transitional system, that is between autonomic and voluntary) seems to be the key to exerting greater control over autonomic functions such as heart rate. That there can be significant EEG correlates of orgasm (like those deep trance states induced by meditation) should not be surprising and has been confirmed by researchers such as Cohen et al [1976] and Sarrel, Foddy, and McKinnon[1977], Mosovich and Tallafero [1954], Godstein [1975] , Graber et al [1985], although replicability of the results has been questioned by Graber, Rohrbaugh, et al [1985]. The apparent conflicts can simply be due to types of signal processing methods used [Banks, 1990], the amount of noise in the signal and also to the actual variations among the subjects. Correlated firing of neurons, a pattern similar perhaps to alpha waves must have some useful neuronal/physiological function and perhaps helps in formation of new memories by enervating the pathways, similar in some sense to the refresh cycle of dynamic RAM in computers. In another sense it could be analogous to a *reset mechanism*, that is, theta rhythms are search pathways and not the same as the alpha waves in which "natural" thought patterns are activated. This electrical storm of particular parts of the brain in which correlated or coherent firings of masses of neurons may be related in some ways to the effects of mediation or even the partial complex seizures (i.e. partial epilepsy). Orgasm might reset the erratic firing patterns of neurons and thus accomplish what meditation does when it produces the alert external concentration mode, and thus cease the "internal dialogue" of Castaneda. Whereas modern society expects concentration in many areas and deep thought, not alertness as in Zen Buddhism of "Be here now." Furthermore things such as premature ejaculation are probably best explained in terms of the rapid change of the response surface due to psychological effects. Therefore what we really want is to have yet another set of equations in which the parameters are functions of yet more psychological variables. This part is much more difficult since it's difficult to even make up let alone measure such variables. [See for example Hubey1996]. Psychology itself is little understood in the scientific sense, say of physics, due to the immense complexity of the task, and what the future will bring can perhaps be best summed up as in:

> It should be noted that experimental support for the cognitive arousal approach, as a general theory of emotion, is mixed. Schachter and Singer's original emphasis on a"a general pattern of sympathetic excitation" [1962, p.380] as the energizing force for most or all emotional experiences has not been strongly supported [Marshall & Zimbardo, 1979; Maslech, 1979]. However the hypothesis that arousal from an extraneous source can intensify emotional experiences in some situations has led to some interesting findings. [excitation transfer of Cantor, Zillmann, and Bryant]. [Rosen & Beck, 1988:28]

## References

Abraham, R., and C. Shaw (1992) *Dynamics: The Geometry of Behavior*, Addison-Wesley, Menlo Park, CA.

Arnold, L. and R.Lefever (1985) *Stochastic Nonlinear Systems*, Springer-Verlag, New York.

Bancroft, J.H. (1983) Human sexuality and its problems. New York, Churchill-Livingstone.

Banks, P, (1990) Signal Processing, Image Processing and Pattern Recognition, Prentice-Hall, Englewood Cliffs.

Barlow, D.H. (1986) Causes of sexual dysfunction: The role of anxiety and cognitive interference. Journal of Consulting and Clinical Psychology, 54, 140-157.

Braun, M. (1983) *Differential Equations and Their Applications*, Springer-Verlag, New York.

Brindley, G.S. (1983) Cavernosal alpha-blockade: A new technique for investigating and treating erectile impotence, British Journal of Psychiatry, 143, 332-337.

Brooks, D. and E.Wiley (1986) *Evolution as Entropy: Toward a Unified Theory of Biology*, University of Chicago Press.

Butkov, E. (1968) *Mathematical Physics*, Addison-Wesley, Reading, MA.

Byrne, D. (1977) The Imagery of Sex, in J. Money & H. Musaph (Eds.) Handbook of sexology (pp.327-350) Amsterdam, Elsevier.

Byrne, D. (1983) The antecedents, correlates, and consequents of erotophobia-erotophilia, in C. Davis (Ed.) Challenges in sexual science (pp.53-75) Lake Mills, IA, Graphic.

Byrne, D. (1986) The study of sexual behavior as a multidisciplinary venture, in D. Byrne & H. Kelly (Eds) Alternative approaches to the study of sexual behavior (pp. 1-12) Hillsdale, NJ, Erlbaum.

Byrne, D. , Jazwinski, C., DeNinno, J, & Fisher, W.A. (1977) Negative sexual attitudes and contraception, in D.Byrne & L.A. Byrne (Eds.) Exploring human sexuality (pp. 246-261), New York, Harper & Row.

Cantor, J.R., Zillmann, D., & Bryant, J. (1975) Enhancement of experienced sexual arousal in response to erotic stimuli through misattribution of unrelated residual excitation, Journal of Personality and Social Psychology, 32, 69-75.

Casdagli, M. and S. Eubank (1992) *Nonlinear Modeling and Forecasting*, Addison-Wesley, Reading, MA.

Cohen, H.D., Rosen, R.C. & Goldstein, L. (1976) Electroencephalographic laterality changes during human sexual orgasm, Archives of Sexual Behavior, 5, 189-199.

Cushing, J. (1975) *Applied Analytical Mathematics for Physical Scientists*, John Wiley & Sons, New York.

Davidson, J.M. (1980) The psychobiology of sexual experience, in J.M. Davidson & R.J. Davidson (Eds), The psychobiology of consciousness (pp.271-332), New York, Plenum Press.

Davis,A. (1962) *Introduction to Nonlinear Differential and Integral Equations*, Dover, New York.

Dennett, D. C. (1991) *Consciousness Explained*, Little, Brown and Company, Boston.

Devaney, R. (1987) *Introduction to Chaotic Dynamical Systems*, Addison-Wesley, New York.

Durden-Smith, J. and D. DiSimone (1983) *Sex and the Brain*, Warner Books, New York.

Eccles, J. (1989) *Evolution of the Brain: Creation of the Self*, Routledge, New York.

Fakras, G.M., Sine, L.F., & Evans, I.M. (1979) The effects of distraction, performance demand, stimulus explicitness, and personality on objective and subjective measures of male sexual arousal, Behavior Research and Therapy, 17, 25-32.

Ferguson, M.(1973) *The Brain Revolution: Frontiers of Mind Research*, Bantam Books, New York.

Gardiner, C.W. (1983) *Handbook of Stochastic Methods*, Springer-Verlag,New York.

Geer, J.H., & Fuhr, R.(1976) Cognitive factors in sexual arousal: The role of distraction, Journal of Consulting and Clinical Psychology, 44, 238-243.

Gellhorn, E.(1957) Autonomic imbalance and the hypothalamus, Minneapolis:University of Minnesota Press.

Goldstein, L. (1975) Time domain analysis of the EEG: The integrative method, in G. Dolce & H. Kunkel (Eds) CEAN-Computerized EEG analysis (pp.251-270), Stuttgart: Gustav Fisher.

Graber, B., Rohrbaugh, J.W., Newlin, D.B., Varner, J.L., & Ellingson, R.J. (1985) EEG during masturbation and ejaculation, Archives of Sexual Behavior, 14, 491-503.

Guastello, S.(1995) *Chaos, Catastrophe, and Human Affairs*, Lawrence Erlbaum Associates, Mahwah, NJ.

Guckenheimer, John and P. Holmes (1983) *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York.

Hooper,J. and D. Teresi (1986) *The 3-Pound Universe*, Tarcher, Los Angeles.

Hubey, H.M. (1997) *The Diagonal Infinity: Problems of Multiple Scales*, to be published, World Scientific.

Hubey, H.M. (1995), *Intelligence, Statistics, Infinity and Multiple Scales*, submitted to a SNDS.

Hubey, H.M. (1994) *Mathematical and Computational Linguistics*, Mir Domu Tvoemu, Moscow, Russia.

Hubey, H.M. (1991) Catastrophe Theory and Human Sexual Response, presented at the Third International Symposium on Systems Research, Informatics, & Cybernetics, Baden-Baden, Germany, August 12-18.

Hubey, H.M. (1979) *Deterministic and Stochastic Behavior of Continuous Production Systems*, Master's Thesis, New Jersey Institute of Technology.

Jackson, E.,A. (1991) *Perspectives on Nonlinear Dynamics*, Cambridge University Press, New York.

Jazwinski, (1970) *Stochastic Processes and Filtering Theory*, Academic Press, New York

Jerison, H.J. (1973) *Evolution of the Brain and Intelligence*, Academic Press, New York.

Jones, S. et al (eds)(1992) *Cambridge Encyclopedia of Human Evolution*, Cambridge University Press, Cambridge.

Kaplan, H.S. (1977) Hypoactive sexual desire, Journal of Sex and Marital Therapy, 3, 3-9.

Kaplan, H.S. (1979) Disorders of sexual desire, New York, Brunner/Mazel.

Karacan, I. (1976) Impotence: Psyche vs soma, Medical World News, 17, 28.

Karacan, I., Aslan, C., & Hirskhowitz, M. (1983) Erectile mechanisms in man, Science, 220, 1080-1082.

Kojima, Ken-ichi (1970) *Mathematical Topics in Population Genetics*, Springer-Verlag, New York.

MacLean, P.D. (1973) *A Triune Concept of the Brain and Behavior*, University of Toronto Press, Toronto.

Marshall, G.D., & Zimbardo, P.G. (1979) Affective consequences of inadequately explained physiological arousal, Journal of Personality and Social Psychology, 37, 970-988.

Maslach, C. (1979) Negative emotional biasing of unexplained arousal, Journal of Personality and Social Psychology, 37, 953-969.

Masters, W. and V. Johnson (1966) *Human Sexual Response,* Little and Brown.

Minorsky, N. (1962) *Nonlinear Oscillations,* Van Nostrand.

Moreault, D., & Follingstadt, D.R. (1978) Sexual fantasies of females as a function of sex guilt and experimental response cues, Journal of Consulting and Clinical Psychology, 46, 1385-1393.

Mosher, D.L., & O'Grady, K.E. (1979) Sex guilt, trait anxiety, and females' subjective sexual arousal to erotica, Motivation and Emotion, 3, 235-249.

Mosovich, A., & Tallafero, A. (1954) Studies of EEG and sex function at orgasm, Diseases of the Nervous System, 15, 218-220.

Mould, D.E. (1980) Neuromuscular aspects of women's orgasms, Journal of Sex Research, 16, 193-201.

Nayfeh, A. (1981) *Introduction to Perturbation Techniques*, Wiley-Interscience, New York.

Nayfeh, A. (1973) *Perturbation Methods*, Wiley-Interscience, New York.

Ornstein,R. (1975) *Psychology of Consciousness,* Penguin Books, New York.

Ornstein, R., R. Thompson, and D. Macaulay (1984) *The Amazing Brain*, Houghton-Mifflin, Boston.

Ott, Edward (1993) *Chaos in Dynamical Systems*, Cambridge University Press, Cambridge.

Restak, R. (1979) *The Brain: The Last Frontier*, Warner Books, New York.

Robinson, P. (1976) The modernization of sex, New York, Harper & Row.

Rose, S. (1976) *The Conscious Brain*, Vintage Books, New York.

Rosen, R. (1988) *Patterns of Sexual Arousal: psychological processes and clinical applications*, Guilford Press.

Rosen, R.C.,. & Leiblum, S.R. (1987) Current approaches to the evaluation of sexual desire disorders, Journal of Sex Research, 23, 141-162.

Roughgarden, J. (1979) *Theory of Population Genetics And Evolutionary Ecology: An Introduction*, MacMillan, New York.

Sagan, Carl (1977) *Dragons of Eden: Speculations on the Evolution of Human Intelligence*, Ballantine Books, New York.

Sarrel, P.M., Foddy, J., & McKinnon, J.B. (1977) Investigation of human sexual response using a casette recorder, Archives of Sexual Behavior, 6, 341-348.

Schachter, S., & Singer, J. (1962) Cognitive, social, and physiological determinants of emotional state, Psychological Review, 69, 379-397.

Schmidt-Nielsen, K. (1984) *Scaling: Why is Animal Size So Important?*, University of Cambridge Press, Cambridge.

Schroeder, M. (1991) *Fractals, Chaos, Power Laws*, Freeman and Company, New York.

Scientific American, (1993) *Mind and Brain*, Freeman, New York.

Searle, J. (1984) *Minds, Brains, and Science*, Harvard University Press, Cambridge, MA.

Searle, J. (1984) *Minds, Brains, and Science*, Harvard University Press, Cambridge, MA.

Sherfey, M.J. (1966) The nature and evolution of female sexuality, New York, Random House.

Skinner, B.F. (1974) *About Behaviorism*, Vintage Books, New York.

Soong, T.T. (1973) *Random Differential Equations in Science and Engineering*, Academic Press, New York.

Sprott, J. (1993) *Strange Attractors*, Henry Holt & Co., NY.

Stevens, K., (1989) *On the quantal nature of speech*, Journal of Phonetics, 17, pp.3-45.

Strogatz, S. (1994) *Nonlinear Dynamics and Chaos*, Addison-Wesley, Reading, MA.

Temple, C. (1993) *The Brain*, Penguin Books, London.

Thom, R. (1975) *Structural Stability and Morphogenesis; an outline of a general theory of models*, W.A.

Benjamin, Reading, MA.

Wang, Wm. [Ed] (1986) *The Emergence of Language: Development and Evolution*, Freeman, New York.

Weiss, H.D. (1972) The physiology of human erection, Annals of Internal Medicine, 76, 793-799.

Woodcock, A. and M. Davis, (1978) *Catastrophe Theory*, Avon, New York.

Zeeman, E. (1977) *Catastrophe Theory: Selected Papers 1972-1977*, Addison-Wesley.

Zurek, W. (ed) (1995) *Complexity, Entropy, and the Physics of Information*, Addison-Wesley, New York.