RUI: Extremal problems related to graph homomorphisms
Impact statement

[redacted]

1 Overview of the Research Environment

Montclair State University. With over 18,000 students, Montclair State University (MSU) is the second largest public university in New Jersey, and has a student population that is 10% African-American and 18% Hispanic. The University was ranked in the top 2% (84 out of over 4800) nationally for awarding undergraduate degrees to minority students [3]. MSU offers over 250 majors, minors, and concentrations with STEM majors housed in the College of Science and Mathematics. In the last ten years, the University has experienced major growth as evidenced by the student population growing from 13,000 in 1999, the hiring of many new full-time faculty, and much construction on campus, including dormitories and academic buildings.

The University is strongly invested in the academic success of its students, an investment that is reflected in graduation rates that significantly exceed the average among institutions of the same Carnegie classification and have been consistently increasing over the past five years [1]. It is also important to note that graduate rates at MSU do not vary significantly by ethnicity [4]. In fact, MSU is a leading institution nationally in the awarding of undergraduate degrees to Hispanics and, in particular, in the mathematical sciences [2].

The University places equal value on teaching and scholarly activity for its faculty. For faculty that are active in research, the University offers the Faculty Scholarship Program (FSP) in order to support their research and other scholarly activities. Faculty who are part of program, including PI [redacted] have their teaching assignments reduced by six credits per year, so that their total teaching load is eighteen credits annually. Most courses at the University meet twice a week, and course assignments in the Department of Mathematical Sciences are made so that faculty have large blocks of time that can be devoted to research during the academic year.

Student research is encouraged and promoted by the University. In 2007, Diana Thomas of the Department of Mathematical Sciences started the Student Research Symposium, which has grown steadily over the past four years. It is co-sponsored by the College of Science and Mathematics, the College of Humanities and Social Sciences, and the MSU Chapter of Sigma Xi. Now, nearly every department of the sponsoring colleges is represented by either student presentations or posters. This past spring, of the 60 total speakers at the event, twelve were from the Department of Mathematical Sciences, including a Master's student of PI [redacted]

The College of Science and Mathematics. The College of Science and Mathematics (CSAM) has 102 full-time faculty and is made up of the departments of Biology and Molecular Biology, Chemistry and Biochemistry, Computer Science, Earth and Environmental Studies, and Mathematical Sciences. Faculty research is supported internally from several institutes which operate as part of CSAM, including the Sokol Institute for Pharmaceutical Sciences, the Center for Environmental and Management Analysis, the Passaic River Institute, and the Institute for Sustainability Studies.

In the Fall of 2010, CSAM had 1,977 undergraduate majors, and 410 graduate students. The CSAM undergraduate student population is 25% Hispanic, 13% African-American, and 53% female [4].

CSAM operates a number of programs to prepare students for advanced study and/or careers in the STEM disciplines. The Health Careers Program prepares students from financially and educationally disadvantaged backgrounds for admission to health professions schools and careers in the sciences. CSAM is also the Northern New Jersey hub for the Garden State Alliance for Minority

Progress (a consortium funded under the NSF Louis Stokes Alliance for Minority Participation Program). Under this initiative, CSAM is recruiting up to 25 AMPS scholars, who are to be freshman, sophomores, or transfer students, in addition to seven junior or senior level student mentors. Undergraduate research is supported by internal grants awarded by the Dean's office for summer salary and travel to scientific meetings for student presentations.

The NSF-funded GK-12 Fellows in the Middle program provided stipends and other support for STEM Master's degree students. In addition to undertaking research with a member of the faculty in CSAM, the students worked closely with local middle school teachers to enrich STEM instruction. The research of the fellows was expected to produce a Master's thesis. Over the past two years, PI [redacted] has been a research advisor to [redacted] as part of this program.

The Department of Mathematical Sciences. The Department of Mathematical Sciences is the largest in CSAM with 32 full-time faculty. The department has 360 undergraduate majors and 72 graduate students. The undergraduate population is 21% Hispanic, 6% African-American, and 53% female [4]. The department is made up of four divisions: Pure and Applied Mathematics, Mathematics Education, Physics, and Statistics. The University has consistently hired new faculty in the department, including adding four new faculty this year. These new faculty are expanding the breadth of research activity in the department and helping to contribute to a more active research culture.

The Department of Mathematical Sciences offers Bachelor's degrees in Mathematics with concentrations in Discrete Applied Mathematics and Mathematics of Finance in addition to a Teacher Certification option. In Physics, the department offers a degree with a concentration in Astronomy and another Teacher Certification option, along with a combined B.S./M.S. program in Statistics. Minors in mathematics and physics are also available. The Mathematics major is centered around a required core of courses that provide a strong background in mathematical thinking before students undertake their upper-level classes.

Graduate programs in mathematics and statistics lead to the Master's degree in Mathematics with four possible concentrations: Mathematics Education, Pure and Applied Mathematics, Statistics, and Computer Science. The department also offers a Master's degree in Statistics. The Mathematics Education division offers an Ed.D. in Mathematics Pedagogy, but the department does not offer a Ph.D., and so faculty do not teach Ph.D.-level courses.

Courses taught by the faculty in the Department of Mathematical Sciences have fewer than 40 students and are, in general, taught with a great deal of interaction between the students and the lecturer. Computing is integrated into the courses with the use of mobile carts of laptops that allow the faculty to set up computer labs in any classroom. Also, many of the classrooms used by the department are equipped with a desktop computer connected to a projector for use in demonstrations by faculty.

2 Role of Research in the Department

The department's 32 faculty are active in research and professional activities, and research opportunities are available for all students including undergraduates.

The areas in which the department does the majority of its research are fluid dynamics (in particular, ferrofluids), mathematical biology, financial mathematics, dynamical systems, and discrete mathematics. Faculty in the department often attend and give seminars at the many research institutions in the surrounding area. Also, the department hosts a seminar to which external speakers are invited. Recent speakers have come from Rutgers/DIMACS, the US Naval Research Laboratory, Dartmouth, University of California-San Diego, and Virginia Tech. Every semester, the college hosts a CSAM Seminar in the Mathematical Sciences. Recent talks were given by Steven

Heymsfield from Merck Pharmaceuticals, Jamie Radcliffe from the University of Nebraska-Lincoln, and Lars Winther Christensen from Texas Tech University.

Undergraduate research is a growing and integral part of the department's activities. Dr. Aihua Li has been awarded two Council for Undergraduate Research in Mathematics (CURM) grants, the second of which was also awarded to Dr. David Trubatch. Both Dr. Lora Billings and Dr. Bogdan Nita have had numerous projects with undergraduates, some of which have led to publications. Last year, Dr. Diana Thomas had a paper which was partially written with an undergraduate student, published with an undergraduate at MSU. In addition to these publications, faculty at MSU have developed and run both local and national conferences and sessions for undergraduate students to present their research. As mentioned above, Dr. Thomas runs the Annual Student Research Conference here at MSU which gives students from the entire university the opportunity to present their research in both poster sessions and short talks. Dr. Thomas also helps to organize the annual undergraduate research poster session at the AMS-MAA Joint Mathematical Meetings. MSU students have won awards at national undergraduate conferences. For example, recent graduate Elizabeth Arrango won the best speaker award at her session at the 2008 MAA MathFest Student Paper session and Cihan Karabulut was a prize winner at the 2008 undergraduate poster session at the Joint Meetings.

The Department of Mathematical Sciences is becoming more successful in producing students that go on to pursue Ph.D.s at larger research institutions. This has been due to a wide range of programs that offered to both undergraduate and Master's level students. The student-run math club has recently been attracting many students and has started a peer-tutoring program. Also, a team for the Putnam examination was formed by PI [redacted] in the fall of 2009 with weekly problem solving sessions, and continues this year. One recent and one current undergraduate, Cihan Karabulut and Kaitlyn Murphy, have participated in REUs at SUNY-Potsdam and Iowa State University, respectively. Three students interested in discrete mathematics have recently gone on to pursue Ph.D.s: Thomas Savitsky at George Washington University, Jonathan Ballone at the University of Delaware, and Scott Binski at the University of Memphis. These students were mentored by PI [redacted] while students at MSU.

Recent grants which demonstrate the department's commitment to math education and training and cross-disciplinary research include these two initiatives:

Montclair State University received a $400,000 grant from the New Jersey Department of Labor and Workforce Development for the Traders to Teachers program, an accelerated three-month program which allows displaced financial services employees to obtain public school certification to teach mathematics. The newly-funded program is a collaboration between the College of Education and Human Services and the College of Science and Mathematics. Dr. Helen Roberts, Chair of Department of Mathematical Sciences, is the co-Principal Investigator for this program. Applicants are required to pass a math aptitude test and complete an application process that includes spending one day serving in schools. Successful applicants will spend three months in an intensive, full-time program learning mathematics and how to teach it, and spending one day each week observing and beginning to teach mathematics in a middle school or high school. Upon conclusion of the program, candidates will also have to pass the required content knowledge Praxis examination. During the first year of teaching, the program will provide intensive induction support and mentoring.

Dr. Lora Billings has just been awarded a three year NIH grant, Multi-scale Modeling of Infectious Diseases in Fluctuating Environments – Prediction and Control for $799,310, along with her Co-Principal Investigators, Dr. Derek Cummings at Johns Hopkins University and Dr. Leah Shaw at the College of William and Mary. Dr. Billings is part of a research team that includes scientists from both the life sciences and mathematical sciences communities. Through this collaborative research, the project aims to develop new mathematical models and methods that predict and

prevent outbreaks of infectious diseases. The team will examine the dynamics of disease spread in fluctuating environments modeled at various population scales. The objective of this research grant is to develop new mathematical models of infectious disease transmission that will effectively capture the impact of stochasticity of dynamics and lead to more effective control. The group will study the dynamics of disease spread in fluctuating environments modeled at various population scales. The research will lead to great insight into the mechanisms that allow a disease to successfully propagate in a population as well as new mathematical tools to analyze stochastic systems. This grant is funded by an initiative to support innovative mathematics needed to solve important biological problems. Considerable excitement has been generated in the research community about the impact of interdisciplinary efforts combining mathematical fields such as nonlinear analysis, stochastic dynamics, and network theory, with systems biology approaches such as population dynamics, epidemiology, and immunology.

Dr. David Trubatch, along with Dr. Philip Yecko, Dr. Lora Billings, and Dr. Bogdan Nita of the Department of Mathematical Sciences and Dr. Chunguang Du of the Department of Biology and Molecular Biology, was recently awarded a Major Research Instrumentation grant from the NSF for the purchase of a high-performance computing cluster for CSAM. The cluster will help research in CSAM that has applications in diverse areas including stochastic metapopulation disease models, functional genomics of the maize genome, liquid atomization modeling, magnetic fluid dynamics modeling, and imaging by inverse scattering. Students will be involved in many of these problems and they will be trained to use the cluster, a skill that should serve them well in their future careers. PI [redacted] is mentoring a student, [redacted] who is setting up an experiment to run on the cluster that will test small cases of a conjecture related to graph homomorphisms.

3 Impact of the Proposed Research

As seen above, the Department of Mathematical Sciences has a very active undergraduate research program already in place. This project aims to augment this program by supporting undergraduate and graduate research projects with the goal of preparing these students for doctoral programs at research institutions.

The proposal aims to expose students to many aspects of proposed mathematical research in order for them to see themselves as members of the mathematical community. This, in turn, will encourage them to pursue careers in the mathematical sciences. Many students at MSU are the first in their families to attend college, and so this opportunity will open even further doors by introducing academic research to them. The time spent with PI [redacted] as a mentor will help with their mathematical maturity as well, which will increase their chances of future success in graduate studies.

The proposed funding for undergraduate assistants will allow students who otherwise may need to find jobs outside of school to spend their time engaged in mathematical research. Many students at MSU fund their own education and living expenses, and even help support their families, through outside work. The undergraduate research proposed in the project would also fit in with the department's goal of sending more students to REUs and onto graduate work at major research institutions. As mentioned in the project summary, the PI has had experience with a successful undergraduate research project while at the University of Nebraska-Lincoln that led to a publication in *Discrete Mathematics*. In addition, the PI is now working with an undergraduate [redacted] on a project related to graph homomorphisms which should result in a research publication to be submitted sometime in the next year. The PI is also just starting a project in graph theory with an undergraduate [redacted] as part of the SHIP program, an honors program in CSAM. This project proposes to support the work of two undergraduate students as they undertake research

involving graph homomorphisms and related questions. An effort will be made to attract students in underrepresented minorities, which, as noted above, make up a substantial part of the student population at MSU and in the Department of Mathematical Sciences. Further, the students will be funded to travel to conferences to present their research which will hopefully lead to new connections with research institutions where they could pursue graduate degrees.

In addition to the undergraduate research in the project, there will also be a graduate student at the Master's level supported by the proposal. The graduate student will work with the PI over the summer on both research and preparation for mentoring the undergraduate researchers during the academic year. The PI has had a two Master's students successively defend their theses, and currently has two graduate students working with him in graph theory and combinatorics, both of whom will write a thesis over the next academic year. The two current students [redacted] have submitted a paper with results obtained over the past summer with PI [redacted].

With the help of the FSP program, faculty at MSU are able to maintain research activity during the academic year. However, the summer remains an important time to advance research programs. The proposed summer funding for PI [redacted] will help to organize and develop the research program proposed in the project description. This will help to ensure the success of the undergraduate and graduate students involved in the project. Also, it will certainly help the PI to remain connected to the discrete mathematics community as a whole.

While there will be weekly meetings involving the PI and both the undergraduate and graduate students involved with the project to exchange ideas and do research together, starting a discrete mathematics seminar will help to integrate the project with the department as a whole. Even students not involved with the program will be exposed to mathematical research and will hopefully see their peers' participation as an inspiration. Students involved with the project will be expected to give talks at the seminar to report on their results and also to practice for conference talks in the future. By seeing these students report on their results, the hope is to get other students in the department interested in research and then to report on results of their own. Also, some travel money will be used to bring in external speakers to the discrete mathematics seminar (and, perhaps, the department seminar), which will expose all students to research they would not otherwise see.

References

[1] Susan Cole, *President's Address to the University Community,* http://www.montclair.edu/president/news/article.php?ArticleID=3675, April 2009.

[2] Editors, *Top 100: Undergraduate Degree Producers,* Diverse Issues in Higher Education **24** (2007), 24-29, 31-45, 47-61.

[3] Editors, *Diverse Issues in Higher Education, Top 100.* http://diverseeducation.com/top100.

[4] Office of Institutional Research, *Montclair State University Fact Book 2010-2011*, Technical Report, Montclair State University, 2010.